

Performance of a MC-CDMA System in a Rayleigh Fading Channel, in the Presence of Channel Estimation Errors

D.N. Kalofonos[†], M. Stojanovic, J.G. Proakis
Northeastern University
Communications and Digital Signal Processing (CDSP) Center
ECE Department, 409 Dana Research Building
360 Huntington Ave.
Boston, MA 02115
Tel : (617) 373-4159, Fax: (617) 373-8970
e-mail: dimitris@cdsp.neu.edu

Abstract

Multi-Carrier CDMA (MC-CDMA) is emerging as a promising multiple access scheme, which combines good performance and high spectral efficiency. Most of the papers that examine the performance of the detection schemes for this system, make the assumption of perfect knowledge of the fading channel, which can be considered accurate in the case of slowly fading channels. In this paper we consider the case when, although the channel is not changing during a MC-CDMA symbol interval, it is allowed to vary during successive symbols, thus causing channel estimation errors. We consider the use of the Kalman filter for channel tracking, which gives the optimal MMSE estimates for the channel coefficients, provided that they are described by a Gauss-Markov state-space model, which is assumed known. We show that as the fading becomes faster, the variance of the estimation errors increases, and we investigate its effect on the performance of a MC-CDMA detector. The paper concludes by discussing the implications of the channel estimation errors on the selection of the system design parameters.

1 Introduction

The increasing demand of high data rate wireless communication systems, driven by the introduction of multimedia applications for transmission through radio channels, has generated the need for highly bandwidth efficient multiple access schemes, which at the same time demonstrate robustness against the hostile nature of the broadband radio channel. One such scheme is Multi-Carrier CDMA (MC-CDMA), which appears as a candidate for broadband wireless communication systems [1].

From the several schemes combining Multi-Carrier Modulation (or OFDM) with Direct Sequence Spread Spectrum (DS-SS) that have been proposed in the literature, the one proposed in [2], [3], [4], has attracted the largest research interest so far. The basic idea of this scheme is to divide the available bandwidth into a large number of narrow subchannels, and spread each data symbol in the frequency domain by transmitting all the chips of a spread symbol at the same time, but in different orthogonal subchannels. Since the chips of all the symbols that form a multi-carrier block overlap in time, even high data rate information can be transmitted using a large MC symbol duration T , which drastically reduces ISI, allows for approximately flat fading in each subchannel, and combats the frequency selective fading of the channel by introducing a large degree of frequency diversity. The focus of this paper is on this MC-CDMA scheme.

[†]Corresponding author

Different detection schemes have been proposed and analyzed for this MC-CDMA system. The synchronous case, which corresponds to the case when all users transmit in perfect synchronism, is more analytically tractable, and has received considerable research attention [5], [6]. Most of the papers that investigate the performance of the MC-CDMA detectors make the assumption that the multiplicative complex channel coefficients, which describe the effect of the frequency selective channel, are perfectly known to the receiver. This assumption can be considered realistic, when the fading rate of the channel is very slow, so that the coherence time of the channel $(\Delta t)_c$ spans several MC symbols, thus allowing for channel estimates containing negligible errors. However, as the channel fading becomes faster, the channel estimates become less accurate, and the errors have an increasingly significant effect on the performance of the MC-CDMA detectors.

In this paper we investigate the impact of channel estimation errors on the performance of the Threshold Orthogonality Restoring Correlation (TORC) detector [7], [8], of a MC-CDMA system that operates in a multipath, fast fading, Rayleigh channel. This suboptimal detector was selected because it combines low complexity with relatively good performance. It also admits a closed form performance expression, which demonstrates the effect of erroneous channel estimation on MC-CDMA detectors which use channel information to obtain the data estimates. We consider the case of a channel described by a first order Gauss-Markov state-space model, whose parameters are assumed known. Although this is a simplifying assumption which involves only one parameter (the 3 dB Doppler spread), it is still more realistic than that of a perfectly known channel regardless of the fading rate. The receiver uses the Kalman filter, which is the MMSE estimator for the channel described by the above model, to track the complex channel coefficients.

This paper is organized as follows: In Section 2, a description of the channel model and of the MC-CDMA system is given; in Section 3, the factors affecting the system performance are examined; in Section 4, simulation results are presented; and in Section 5, we give our conclusions.

2 System Description

We consider a multiple access system where N_u users are transmitting simultaneously in a synchronous manner using Walsh-Hadamard orthogonal codes of length N_s . Therefore up to N_s users can transmit at the same time. The n -th multicarrier block symbol (of duration T_b) for user i is formed by taking μ symbols $b_i^1(n), \dots, b_i^\mu(n)$ in parallel, spreading them with the user's spreading sequence $\mathbf{c}_i = [c_{1,i} \dots c_{N_s,i}]^T$, $c_{j,i} = \pm 1$, performing frequency interleaving, and placing the resulting $u_i^1(n), \dots, u_i^{\mu N_s}(n)$ chips into the $N = \mu N_s$ available subchannels, each having width $\Delta f = 1/T_b$, by using an IFFT of size N . After performing a parallel to serial conversion, a guard interval is added, in the form of a cyclic prefix, and the signals of all the users are added and transmitted through the channel. The block diagram of the transmitter is depicted in Figure 1.

In the rest of this paper, for simplicity of notation, we will concentrate only on one of the μ symbols each user transmits, by setting $\mu = 1$, and keeping in mind that the frequency interleaving function still exists. Also, we will consider binary symbols $b_k(n) = \pm 1$, $k = 1, \dots, N_u$ forming the data vector $\mathbf{b}(n) = [b_1(n), \dots, b_{N_u}(n)]^T$, where n is the time index denoting the n -th symbol interval.

The transmitted signal during the n -th MC block symbol period can be written as follows:

$$s(t) = \sum_{k=1}^{N_u} \sum_{l=1}^{N_s} \sqrt{E_c} c_{l,k} b_k(n) e^{j \frac{2\pi l(t-T_G)}{T_b}} \quad (1)$$

where $t \in [nT, (n+1)T]$, $T = T_b + T_G$, T_G is the guard interval chosen to be at least equal to the delay time spread T_m of the channel, and E_c is the energy per chip.

We consider the case of a fast fading, frequency selective, Rayleigh channel, and we assume that the channel is not changing during one MC symbol interval. We allow, however, for variations during successive symbol intervals. Because of the large symbol duration, the fading in each subchannel is approximately flat, and is described by a multiplicative complex channel coefficient $h_l(n)$, $l = 1, \dots, N_s$, which is a Gaussian distributed, complex, discrete time random process. Because of the frequency

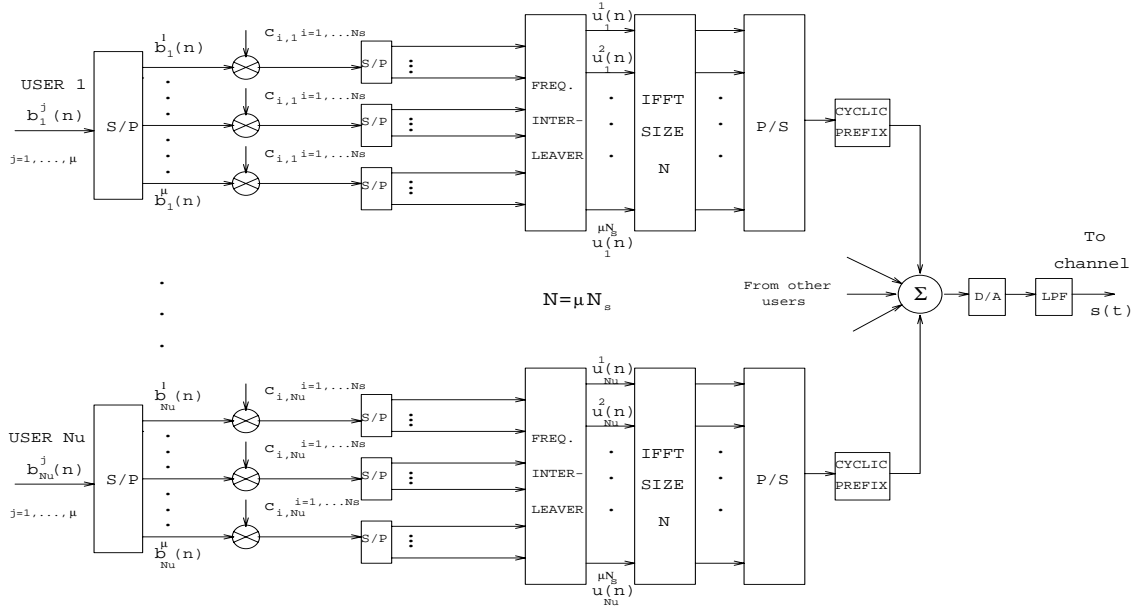


Figure 1: Transmitter block diagram

interleaving function, the channel complex coefficient processes will be considered independent. This is valid in the case when $\mu/T_b > 1/T_m$. We assume that each of these random processes is described by a first order, Gauss-Markov model, of the following form:

$$\text{Process Equation: } h_l(n+1) = fh_l(n) + \chi_l(n), \quad l = 1, \dots, N_s \quad (2)$$

where $\chi_l(n)$ is a zero mean, white Gaussian noise process, with autocorrelation

$$E\{\chi_l(n)\chi_l^*(k)\} = 2\sigma_h^2\delta_{n,k} \quad (3)$$

and $\delta_{n,k}$ is the Kronecker delta. The parameter f corresponds to an exponentially decaying channel time correlation function [9], and is related to the coherence time $(\Delta t)_c$ and the 3-dB Doppler Spread bandwidth B_d as follows:

$$f = e^{-\omega_d T} \quad (4)$$

where $\omega_d = \pi B_d = 2\pi/(\Delta t)_c$.

Because of the existence of a guard interval with duration at least equal to the channel's delay spread, there is no intersymbol interference, and the signal received by user i can be described by the following equation:

$$r(t) = \sum_{k=1}^{N_u} \sum_{l=1}^{N_s} \sqrt{E_c} h_l^{(i)}(n) c_{l,k} b_k(n) e^{j\frac{2\pi l(t-T_G)}{T_b}} + n(t) \quad (5)$$

where $t \in [nT, (n+1)T]$, $h_l^{(i)}(n)$ are the complex channel coefficients which describe the channel between the transmitter and the user i , and $n(t)$ is the AWGN. For simplicity of notation in the rest of the paper, the superscript (i) will be dropped. At the receiver, the signal is sampled at a rate N/T_b , the samples which correspond to the cyclic prefix are discarded, an FFT of size N is performed, and frequency deinterleaving takes place. The vector $\mathbf{r}(n) = [r_1(n), \dots, r_{N_s}(n)]^T$ at the output of the deinterleaver is then fed to the TORC detector, and to the Kalman channel estimator. The vector $\mathbf{r}(n)$ is given in matrix notation by the following equation:

$$\mathbf{r}(n) = \sqrt{E_c} \mathbf{H}(n) \mathbf{C} \mathbf{b}(n) + \boldsymbol{\eta}(n) \quad (6)$$

where $\mathbf{H}(n) = \text{diag}\{h_1(n), \dots, h_{N_s}(n)\}$, matrix $\mathbf{C} = [\mathbf{c}_1 \mid \dots \mid \mathbf{c}_{N_u}]$ is the $N_s \times N_u$ matrix whose columns are the spreading sequences of the users, $\mathbf{b}(n)$ is the data vector of the users, and $\boldsymbol{\eta}(n) = [\eta_1(n), \dots, \eta_{N_s}(n)]^T$ is a vector containing zero mean, uncorrelated complex Gaussian noise samples. The block diagram of the receiver is depicted in Figure 2.

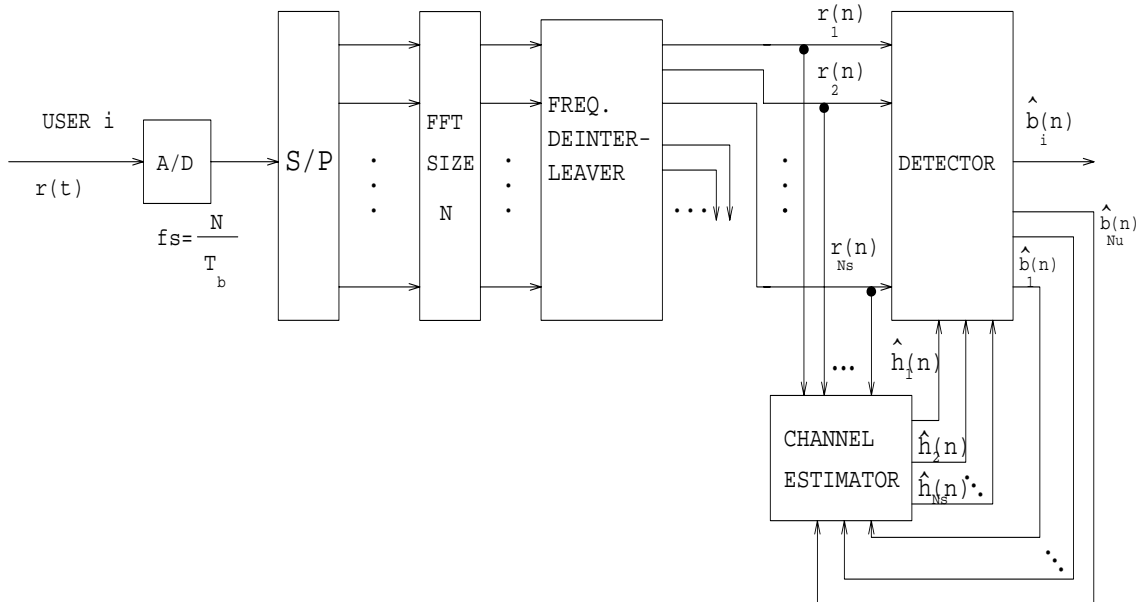


Figure 2: Receiver block diagram

The TORC detector uses the channel estimates to invert the effect of the channel by attempting to invert matrix $\mathbf{H}(n)$. In order to avoid excessive noise enhancement by inverting channel coefficients with very small amplitude, the detector discards the chips which correspond to subchannels with attenuation below a chosen threshold. Then, it correlates the resulting signal with the user's spreading sequence, and performs a threshold decision. The algorithm it uses is the following:

$$\hat{b}_i(n) = \text{sgn}[\mathcal{R}e\{\mathbf{c}_i^T \hat{\mathbf{U}}(n|n-1) \hat{\mathbf{H}}^{-1}(n|n-1) \mathbf{r}(n)\}] \quad (7)$$

where $\hat{\mathbf{H}}(n|n-1) = \text{diag}\{\hat{h}_1(n|n-1), \dots, \hat{h}_{N_s}(n|n-1)\}$ is the estimate of matrix $\mathbf{H}(n)$ by using information available up to the time interval $n-1$, $\hat{\mathbf{U}}(n|n-1) = \text{diag}\{u(|\hat{h}_l(n|n-1)| - h_{THR})\}$, $l = 1, \dots, N_s$, h_{THR} is the selected threshold, and $u(x)$ is the unit step function.

The channel estimates $\hat{h}_1(n|n-1), \dots, \hat{h}_{N_s}(n|n-1)$ are obtained by using the Kalman filter. The estimator's input is the vector $\mathbf{r}(n)$, which represents the measured quantity, and the estimate of the data vector $\hat{\mathbf{b}}(n)$. The equation that describes the measurement vector is derived from (6) by rearranging its terms:

$$\text{Measurement Equation:} \quad r_l(n) = d_l(n)h_l(n) + \eta_l(n), \quad l = 1, \dots, N_s \quad (8)$$

where

$$d_l(n) = \sqrt{E_c} \sum_{k=1}^{N_u} c_{l,k} b_k(n) \quad (9)$$

and $\eta_l(n)$ is the measurement noise process, which is white Gaussian, with zero mean and variance $2\sigma^2$. The Kalman filter produces the channel estimate $\hat{h}_l(n+1|n)$ for the symbol interval $n+1$ by using information available up to the time interval n as follows [10]:

$$\hat{h}_l(n+1|n) = [f - K_l(n)\hat{d}_l(n)]\hat{h}_l(n|n-1) + K_l(n)r_l(n) \quad (10)$$

where the term $K_l(n)$ is the Kalman gain, which is related to the error variance $E_l(n|n-1) = \mathcal{E}\{|h_l(n) - \hat{h}_l(n|n-1)|^2\}$ by the following equation:

$$K_l(n) = \frac{f E_l(n|n-1) \hat{d}_l(n)}{\hat{d}_l^2(n) E_l(n|n-1) + 2\sigma^2} \quad (11)$$

The estimation error variance is calculated in an iterative way:

$$E_l(n|n-1) = f^2 \frac{2\sigma^2 E_l(n-1|n-2)}{\hat{d}_l^2(n-1) E_l(n-1|n-2) + 2\sigma^2} + 2\sigma_h^2 \quad (12)$$

where $\hat{d}_l(n)$ denotes the quantity given in (9), which is obtained by using the estimate of the data vector $\hat{\mathbf{b}}(n)$.

3 System Performance

In this section, the main factors which affect the performance of the system will be identified, and the impact of the channel estimation process will be examined. First we obtain an expression for the decision variable for symbol $b_i(n)$ of user i , $i = 1, \dots, N_u$:

$$v_i(n|n-1) = \mathcal{R}e\{\mathbf{c}_i^T \hat{\mathbf{U}}(n) \hat{\mathbf{H}}^{-1}(n) \mathbf{r}(n)\} \quad (13)$$

We denote by $\hat{\rho}_l(n|n-1)$ and $\hat{\phi}_l(n|n-1)$ the magnitude and the phase, respectively, of the estimate $\hat{h}_l(n|n-1)$, and by $e_l(n|n-1) = h_l(n) - \hat{h}_l(n|n-1)$, the error in the estimation of $h_l(n)$, $l = 1, \dots, N_s$. If we also define $\eta_{lr}(n) = \mathcal{R}e\{\eta_l(n) e^{-j\hat{\phi}_l(n|n-1)}\}$ and $e_{lr}(n|n-1) = \mathcal{R}e\{e_l(n|n-1) e^{-j\hat{\phi}_l(n|n-1)}\}$, the decision variable given in (13) can be rewritten in the following form:

$$\begin{aligned} v_i(n|n-1) &= \sqrt{E_c} \sum_{l=1}^{N_s} u(\hat{\rho}_l(n|n-1) - h_{THR}) b_i(n) + \\ &+ \sqrt{E_c} \sum_{\substack{j=1 \\ j \neq i}}^{N_u} \sum_{l=1}^{N_s} u(\hat{\rho}_l(n|n-1) - h_{THR}) c_{l,i} c_{l,j} b_j(n) + \\ &+ \sum_{l=1}^{N_s} c_{l,i} u(\hat{\rho}_l(n|n-1) - h_{THR}) \frac{\eta_{lr}(n)}{\hat{\rho}_l(n|n-1)} + \\ &+ \sqrt{E_c} \sum_{j=1}^{N_u} \sum_{l=1}^{N_s} u(\hat{\rho}_l(n|n-1) - h_{THR}) c_{l,i} c_{l,j} \frac{e_{lr}(n|n-1)}{\hat{\rho}_l(n|n-1)} b_j(n) \\ &= S_i(n) + I_i(n) + \xi_i(n) + \Delta_i(n) \end{aligned} \quad (14)$$

In equation (14) the first term $S_i(n)$ corresponds to the desired signal, the second $I_i(n)$ to the multiuser interference, the third $\xi_i(n)$ to the noise, while the fourth $\Delta_i(n)$ arises from the channel estimation errors. The interference and noise terms exist even in the case of perfect channel knowledge. Therefore, in the rest of this section the effect of the term $\Delta_i(n)$ will be examined.

The error, $e_l(n|n-1) = h_l(n) - \hat{h}_l(n|n-1)$, is a zero mean Gaussian random variable, and its variance $E_l(n|n-1)$ can be calculated in an iterative way from (12), where it can be seen that the error variance depends on the data vector $\mathbf{b}(n-1)$ through the term $\hat{d}_l^2(n-1)$. In order to eliminate this dependency and to obtain a quasi-steady form of equation (12), we approximate the term $\hat{d}_l^2(n-1)$ by its average value $N_u E_c$. Then, in the steady state, the error variance is given approximately by

$$\tilde{E}_l = \frac{2\sigma_h^2 N_u E_c - 2\sigma^2(1-f^2) + \sqrt{(2\sigma^2(1-f^2) + 2\sigma_h^2 N_u E_c)^2 + 16\sigma^2 \sigma_h^2 N_u E_c f^2}}{2N_u E_c} \quad (15)$$

In Figure 3, numerical evaluations of the above approximate expression, together with simulation results are depicted, which show that the approximate error variance in (15) is smaller than the actual variance by less than 1 dB, for fading rates ($\omega_d T$) between 10^{-4} and 10^{-1} . We notice that

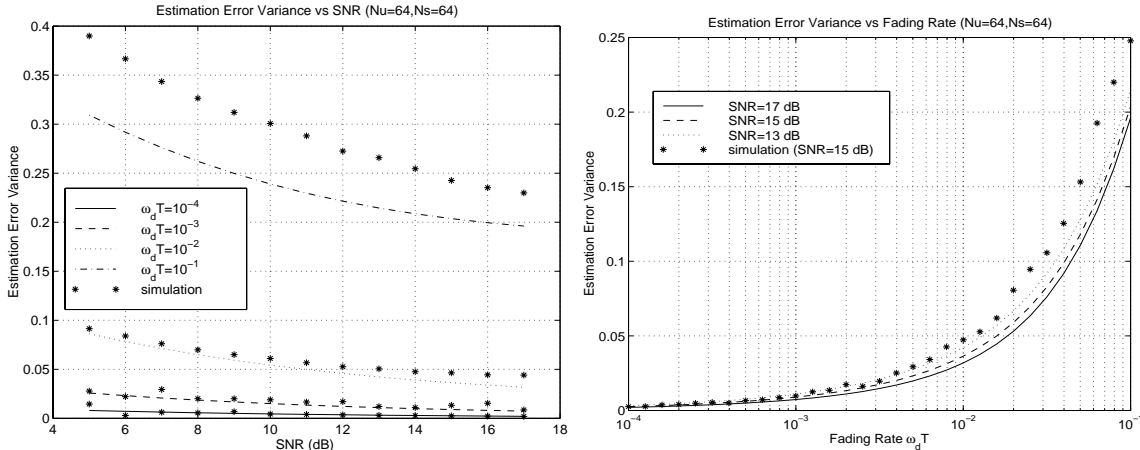


Figure 3: Left: Estimation error variance as a function of SNR for various fading rates. Right: Estimation error variance as a function of the fading rate for various SNR's.

the estimation error variance increases significantly as the fading becomes faster (larger $\omega_d T$). Since the variance of the decision variable (14) increases with increasing error variance, we expect that the performance of the system deteriorates substantially in the case of fast fading channels. The simulation results presented in the next section support this remark.

In the above figures, and throughout this paper, the average SNR per bit γ_b is defined as follows:

$$\gamma_b = \frac{N_s E_c \mathcal{E}\{|h_l(n)|^2\}}{2\sigma^2} = \frac{N_s E_c \sigma_h^2}{(1-f^2)\sigma^2} \quad (16)$$

In (16) the last equality holds because each of the channel coefficients $h_l(n)$, which is described by the process equation (2), is Gaussian with zero mean and with variance [10]:

$$\text{Var}\{h_l(n)\} = \mathcal{E}\{|h_l(n)|^2\} = 2\sigma_h^2 \left[f^{2n} + \frac{1-f^{2n}}{1-f^2} \right] \quad (17)$$

Clearly in the steady state (large n), $\text{Var}\{h_l(n)\} = 2\sigma_h^2/(1-f^2)$. In all cases we normalize the channel coefficients so that

$$\mathcal{E}\{|h_l(n)|^2\} = 1 \Leftrightarrow \frac{2\sigma_h^2}{1-f^2} = 1 \quad (18)$$

From (18) it is obvious that, in order to keep a constant reference in the definition (16) of SNR, the variance σ_h^2 should be varied accordingly as f changes for different fading rates $\omega_d T$.

Another way of demonstrating the effect of the channel fading rate on the channel estimation process is depicted in Figure 4, where the realizations of the channel and the Kalman filter estimate are shown, in the case of a fast and of a slowly fading channel.

4 Simulation Results

In this section we give several examples that illustrate the performance of the MC-CDMA system under consideration, by presenting simulation results.

As it was pointed out in the previous section, as the channel fading becomes faster ($\omega_d T = 2\pi T/(\Delta t)_c$ increases), the estimation error and its variance \tilde{E}_l grow larger, thus increasing the variance

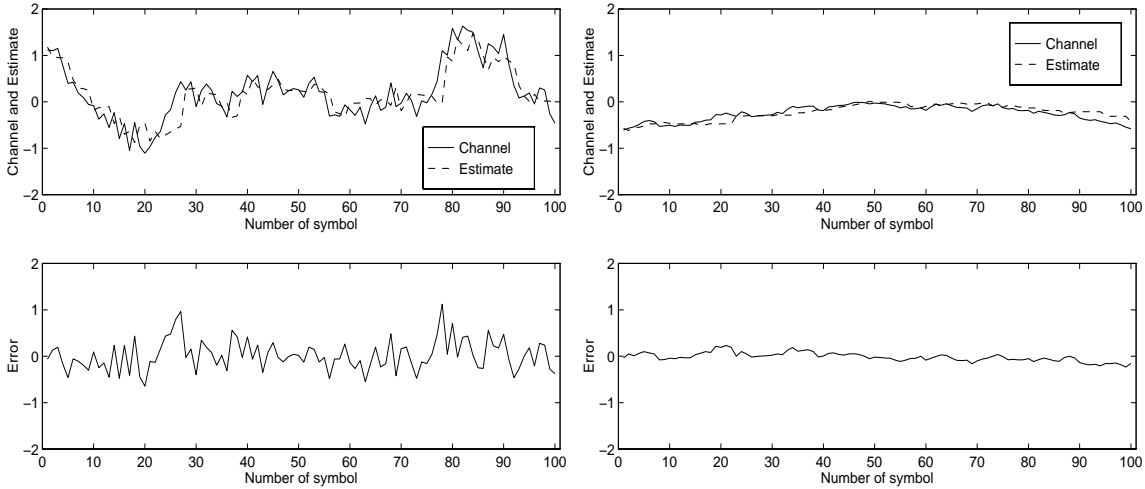


Figure 4: A channel realization and the Kalman estimate for $\omega_d T = 10^{-1}$ (fast fading-left) and $\omega_d T = 10^{-3}$ (slow fading-right).

of the term $\Delta_i(n)$. This, in turn, degrades the performance of the detector, and this is depicted in Figure 5. We can see that for a probability of error 10^{-3} , the performance deteriorates by 1 dB for a fading rate of $\omega_d T = 10^{-3}$, by 3 dB for a fading rate of $\omega_d T = 10^{-2}$, while for very fast fading, $\omega_d T = 10^{-1}$, the detector performs very poorly.

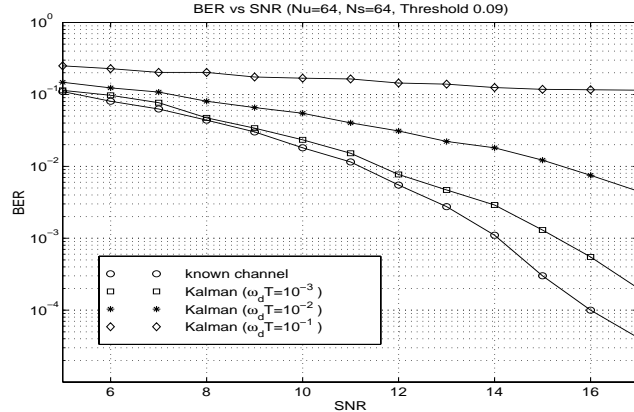


Figure 5: Probability of error as a function of γ_b for fading rates $\omega_d T$ between 10^{-3} (slow fading) and 10^{-1} (fast fading). The threshold is chosen $h_{THR} = 0.09$.

It has been shown [5], [8], that there is an optimal selection of the threshold h_{THR} for each SNR. In Figure 6, the dependence of the probability of error on the threshold h_{THR} is depicted. We notice that as the channel fading becomes faster, the optimal threshold h_{THR} increases. An interesting question for further investigation would be to find an adaptive way of selecting the threshold h_{THR} , depending on the observed SNR and the channel fading rate.

Another implication of the channel estimation process is on the selection of the length of the spreading sequences N_s , and the number of subchannels $N = \mu N_s$. We expect that increasing N_s will improve the performance of the system, by introducing a larger degree of frequency diversity. This is, indeed, true in the case when the channel is perfectly estimated at the receiver, as it can be seen in Figure 7. For a fixed total bandwidth $W = \mu N_s \Delta f$, increasing N_s by a factor λ , implies that $\Delta f_{new} = \Delta f / \lambda$, and that the new symbol duration should be $T_{b,new} = 1 / \Delta f_{new} = \lambda T_b$. Therefore,

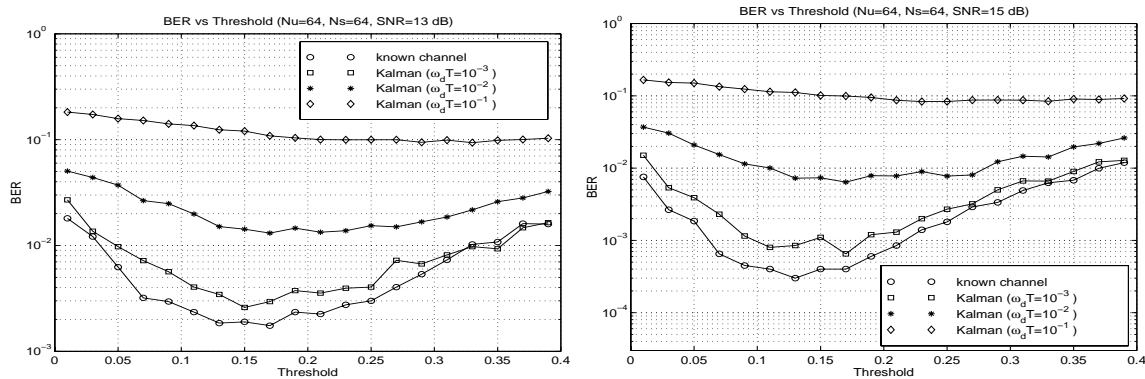


Figure 6: BER vs threshold for SNR=13 dB (left) and SNR=15 dB (right), for ω_dT between 10⁻³ (slow) and 10⁻¹ (fast).

the fading rate ω_dT is also increased λ times, thus affecting adversely the performance of the system. This can be seen in Figure 7, where, although in the case of perfect channel knowledge (very slow fading) the system with N_s = 64 outperforms the one with N_s = 16, in the case of imperfect channel estimation it performs worse, because it is subject to a fading rate that is four times higher.

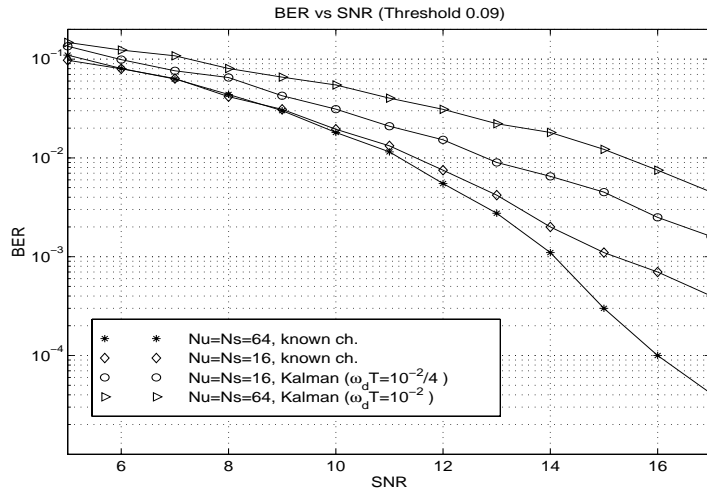


Figure 7: Impact of channel estimation errors on the selection of N_s.

5 Conclusions

In this paper the performance of the TORC detector of a MC-CDMA system operating in a fast fading, multipath Rayleigh channel, in the presence of channel estimation errors was investigated, and an analytical expression for the probability of error was derived. The results obtained show that the performance of the system deteriorates significantly, as the fading becomes faster. The optimal threshold for a given SNR also depends on the fading rate, and increases with increasing fading rate ω_dT. It was also found that increasing the degree of frequency diversity by increasing the length of the spreading sequences, might have a negative impact on the performance of the system, depending on the fading rate of the channel. For a given channel coherence time (Δt_c) and total bandwidth W, there is an optimal selection of the number of subchannels.

In the analytical derivation of the performance of the MC-CDMA system considered in this paper, the assumption that the channel is described by a perfectly known, first order, Gauss-Markov model is fundamental. This assumption, however, although more realistic than that of a perfectly known channel, is still too simple, and a higher order model would be more appropriate to describe more practical channels.

We expect that the impact of channel estimation errors on the performance of the MC-CDMA system we considered is representative for most of the MC-CDMA detectors, which use channel information to estimate the user's data symbols.

References

- [1] Morinaga N., Masao, and N. Ryuji K. "New Concepts and Technologies for Achieving Highly Reliable and High-Capacity Multimedia Wireless Communications Systems". *IEEE Communications Magazine*, pages 34–40, January 1997.
- [2] Chouly A., Brajal A., and Jourdan S. "Orthogonal Multicarrier Techniques Applied to Direct Sequence Spread Spectrum CDMA Systems". In *IEEE International Conference on Communications*, pages 1723–1728, 1993.
- [3] Fazel K. "Performance of CDMA/OFDM for Mobile Communication Systems". In *2nd IEEE International Conference on Universal Personal Communications (ICUPC)*, pages 975–979, 1993.
- [4] Yee N., Linnartz J., and Fettweis G. "Multi-Carrier CDMA in Indoor Wireless Radio Networks". In *Proceedings of PIMRC, Yokohama, Japan*, pages 109–113, 1993.
- [5] Muller T., Rohling H., and Grunheid R. "Comparison of Different Detection Algorithms for OFDM-CDMA in Broadband Rayleigh Fading". In *IEEE Vehicular Technology Conference*, pages 835–838, 1995.
- [6] Kaiser S. "Analytical Performance Evaluation of OFDM-CDMA Mobile Radio Systems". In *Proceedings of the first European Personal and Mobile Communications Conference, EPMCC '95, Bologna, Italy*, pages 215–220, November 1995.
- [7] Yee N. and Linnartz J. "Controlled Equalization of MC-CDMA in an Indoor Rician Fading Channel". In *IEEE Vehicular Technology Conference*, volume 3, pages 1665–1669, 1994.
- [8] Kalofonos D.N. and Proakis J.G. "Performance of the Multi-Stage Detector for a MC-CDMA System in a Rayleigh Fading Channel". In *IEEE Global Communications Conference (GLOBECOM '96)*, volume 3, pages 1784–1788, November 1996.
- [9] Stojanovic M., Proakis J.G., and Catipovic J. "Analysis of the Impact of Channel Estimation Errors on the Performance of a Decision-Feedback Equalizer in Fading Multipath Channels". *IEEE Transactions on Communications*, 43(2/3/4):877–885, February/March/April 1995.
- [10] Anderson B.D.O. and Moore J.R. "*Optimal Filtering*". Prentice-Hall, 1979.