On the Performance of Coded Low Spreading Gain DS-CDMA Systems with Random Spreading Sequences in Multipath Rayleigh Fading Channels

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Abstract - **Next generation wireless systems based on Variable Spreading Gain (VSG) DS-CDMA offer variable data-rates while keeping constant the signal envelope and bandwidth. For high data-rate services the SG can be as low as 2 chips per transmitted (coded) symbol, which deteriorates the system performance because it enhances the Inter-Path Interference (IPI). Recently, different Self-Interference Cancelation (SIC) approaches have been examined in uncoded VSG-CDMA systems to combat IPI. In this paper we examine the effect of low SG in practical VSG-CDMA systems, which use Convolutional and Turbo coding and concatenated short orthogonal and long pseudo-random spreading. We show that the performance of the conventional RAKE receiver degrades only moderately for SG as low as 4 chips per transmitted symbol for $BER \geq 10^{-6}$ and for different multipath profiles. In this range of interest the error floor typical to Turbo codes is observed due to the free distance properties of these codes rather than the effect of IPI. This suggests that the increased complexity of SIC schemes is justified only for very high data-rates.**

I. INTRODUCTION

Variable Spreading Gain CDMA (VSG-CDMA) [1] has been adopted in next generation CDMA systems (e.g. IS-2000, W-CDMA) in order to accommodate different data-rates while keeping constant the signal envelope and bandwidth. A common assumption in DS-CDMA systems with RAKE receivers is that because of the large SG the effect of the Inter-Path Interference (IPI) can be neglected [2]. In high data-rate applications, however, the SG can be as low as 2 chips per transmitted (coded) symbol, which makes the effect of IPI important in the design of next generation systems.

Single-user receiver design and equalization in DS-CDMA systems has recently attracted an intense research interest. Two cases are considered depending on whether short or long spreading codes are used[1], because of the different effect of the multi-path delay spread on the system performance. When short spreading sequences are used, the exact effect of IPI on the performance of the RAKE receiver was examined in [3]. Linear filter receivers were examined in [4], [5] and in a convolutionally coded system in [6]. Decision Feedback Equalization (DFE) was proposed in [7]. When long spreading sequences are used, the effect of IPI on the performance of uncoded systems with RAKE receivers were examined in [8] and for low SG in [9]. Equalization schemes to suppress IPI were proposed in [10] and [11].

In this paper we consider VSG-CDMA systems which in practical applications (e.g. IS-2000, W-CDMA) use concatenated short orthogonal and long pseudo-random sequences, and channel coding. The performance of these systems using the RAKE receiver should be used as a benchmark, against which SIC and equalization schemes are compared. Motivated by the lack of results on the performance of high data-rate/low SG coded VSG-CDMA systems with RAKE receivers, we examine the effect of SG as low as 2 chips per coded symbol on the performance of systems employing Convolutional and Turbo codes. We present a self-interference analysis and we investigate the impact of IPI on coded systems with different multi-path profiles.

The rest of this paper is organized as follows: in Section II we describe the system under consideration and

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[1] Spreading sequences with period equal to one symbol duration are characterized as "short", while if the period is larger than one symbol period (typically hundreds or more symbols) the sequences are characterized as "long".

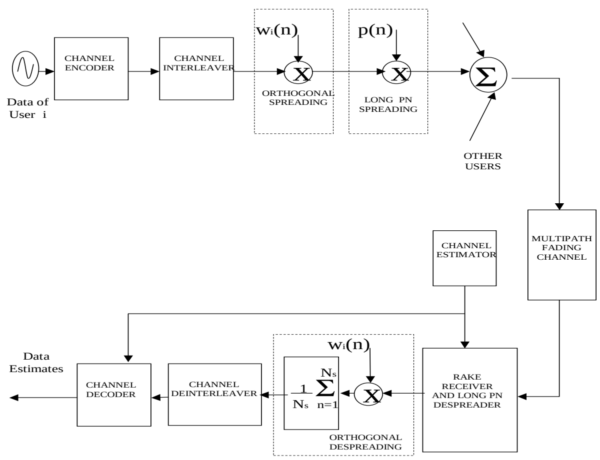

Figure 1: Baseband VSG-CDMA block diagram.

in Section III we present a brief self-interference analysis. In Section IV we present performance results and we explore the need for equalization in coded VSG-CDMA systems with low SG. Finally, in Section V we present the conclusions of this investigation.

II. SYSTEM DESCRIPTION

We consider a simplified baseline system which corresponds to the forward or reverse link of a next generation DS-CDMA system (e.g. IS-2000, W-CDMA). A block of N source binary data symbols $b_i(k) \in \{\pm 1\},\ k = 0, ..., N-1$ of user i are passed through the channel encoder and the encoded binary symbols $d_i(k),\ k = 0, ..., N_f - 1$ are placed into frames of length N_f, where $N_f = N/R$ and R is the overall code rate. These frames are then passed through a channel block interleaver of the same length, and the symbols are spread by using one of the available channelization Orthogonal Variable Spreading Factor (OVSF) sequences $w_i(n),\ n = 0, ..., N_s - 1$ of length N_s (the SG is equal to N_s). For simplicity of notation we consider that the resulting signal is then BPSK spread by using a long PN sequence $p(n)$, although in actual 3G systems QPSK spreading is used. The signals of other users are then added in a synchronous or asynchronous manner, corresponding to the forward and reverse link respectively. In an actual system it is very probable that only one high-data-rate/low-SG user will transmit at each time and that the rest of the users will be low-data-rate/high-SG. Therefore, we assume that their effect can be incorporated in the background noise and in the rest of the paper we drop the user index i. The resulting baseband transmitted signal $s(t)$ has the following form:

$$s(t) = \sqrt{E_c}\, d(t) c(t) \qquad (1)$$

where

$$\begin{aligned} d(t) &= \sum_{k=-\infty}^{\infty} d(k) R_{T_s}(t - kT_s) \\ c(t) &= \sum_{n=-\infty}^{\infty} w(m) p(n) R_{T_c}(t - nT_c) \end{aligned} \qquad (2)$$

$m = n \ mod \ N_s$, T_c and $T_s = N_s T_c$ are the chip and symbol durations respectively, $R_T(t)$ is a unit rectangular pulse of duration T, and E_c is the energy per transmitted chip. The block diagram of the transmitter is depicted in the upper part of Figure 1.

The wireless channel is assumed to be a multipath Rayleigh fading channel with N_p paths modeled by independent zero mean, complex Gaussian distributed path gain processes $a_l(t) = \alpha_l(t) e^{-j\phi_l(t)},\ l = 0, ..., N_p - 1$. The time-variation of the channel is modeled using Jakes' model. The fading rate of the channel is defined as the product $\omega_D T_s$, where ω_D is the maximum Doppler shift. Neglecting the effect of the long term shadowing and path loss, the received signal at the input of the receiver has the following form:

$$r(t) = \sum_{l=0}^{N_p - 1} a_l(t) s(t - \tau_l) + v(t) \qquad (3)$$

where $0 \le \tau_0 < \tau_1 < ... < \tau_{(N_p-1)} \le T_m$ are the path delays, T_m is the maximum delay spread of the channel, and $v(t)$ is zero mean complex AWGN with power spectral density N_0. In the rest of the paper we assume that the path delays are integer multiples of the chip duration, i.e. $\tau_l = \delta_l T_c$, where $\delta_l \in \{0, 1, ..., \lfloor T_m/T_c \rfloor\}$.

At the receiver the signal is first passed through a chip-matched filter and sampled at the chip interval and the samples are fed into a RAKE receiver with N_p fingers. In each finger, the samples are multiplied by the complex conjugate of the corresponding channel coefficients (MRC combining), and then they are correlated with appropriately delayed versions of the PN sequence $p(n)$. The output of the RAKE receiver is formed by adding the outputs of all fingers and taking the real part. The symbol observations $y(k),\ k = 0, ..., N_f - 1$ at the input of the decoder are obtained after performing orthogonal despreading with the sequence $w(n)$. The block diagram of the receiver is depicted in the lower part of Figure 1.

III. SELF-INTERFERENCE ANALYSIS

Self-interference can be assumed negligible only in DS-CDMA systems with large SG. In that case the performance of the "ideal RAKE" receiver is that of an N_p-th diversity combiner using MRC [2]. The effect of non-negligible IPI was examined using a Gaussian approximation in uncoded systems in [8], an approximation which was shown inappropriate for systems with low SG [9], [10]. Without this approximation an exact closed-form expression for the probability of error is intractable even for uncoded systems. In the rest of this section we examine the form of the coded symbol observations and we extract information which helps us understand the performance of coded systems.

The observation of coded symbol $d(k)$ at the input of the decoder has the following form:

$$y(k) = \Re e\{\sum_{l=0}^{N_p-1} a_l^*(k)\frac{1}{N_sT_c}\int_{kT_s+\tau_l}^{(k+1)T_s+\tau_l} r(t)c(t-\tau_l)dt\} \quad (4)$$

In (4), as well as in the rest of this paper, we assume that the channel coefficients $a_l(k)$ are constant during one symbol interval, although they may change over successive intervals, and for simplicity we drop the index k. Substituting (3) and (1) in (4) we can rewrite the observation as a sum of three terms:

$$y(k) = y_s(k) + y_n(k) + y_{IPI}(k) \quad (5)$$

The first term $y_s(k)$ in (5) is the useful signal:

$$y_s(k) = (\sum_{l=0}^{N_p-1} \alpha_l^2)\sqrt{E_c}d(k) \quad (6)$$

The second term $y_n(k)$ in (5) is the AWGN noise:

$$\begin{aligned} y_n(k) &= \Re e\{\sum_{l=0}^{N_p-1} a_l^* \frac{1}{N_sT_c}\int_{kT_s+\tau_l}^{(k+1)T_s+\tau_l} v(t)c(t-\tau_l)dt\} \\ &= \frac{1}{N_s}\sum_{l=0}^{N_p-1}\alpha_l \sum_{n=kN_s+\delta_l}^{(k+1)N_s+\delta_l} w(m)p(n-\delta_l)\eta(n) \end{aligned} \quad (7)$$

where $m = (n - \delta_l)\ mod\ N_s$ and $\eta(n)$ is zero mean AWGN with variance $N_0/2$. Conditioned on the path gains, the noise term has zero mean and variance

$$var\{y_n(k)\} = (\sum_{l=0}^{N_p-1} \alpha_l^2)\frac{N_0}{2N_s} \quad (8)$$

The third term $y_{IPI}(k)$ in (5) corresponds to the IPI:

$$\begin{aligned} y_{IPI}(k) = \Re e\{\sqrt{E_c}\sum_{l=0}^{N_p-1} a_l^* \sum_{\substack{l'=0 \\ l'\neq l}}^{N_p-1} a_{l'}\frac{1}{N_sT_c} \\ \int_{kT_s+\tau_l}^{(k+1)T_s+\tau_l} d(t-\tau_{l'})c(t-\tau_{l'})c(t-\tau_l)dt\} \end{aligned} \quad (9)$$

The IPI in (9) can be rewritten similarly to [8], [9] as the sum of $N_p(N_p-1)/2$ terms of mutual (sum of) interference $Y_{l,l'}(k)$ between the fingers l, l' of the RAKE:

$$y_{IPI}(k) = \sum_{l=0}^{N_p-2}\sum_{l'=l+1}^{N_p-1} Y_{l,l'}(k) \quad (10)$$

The mutual interference between fingers l, l' in general consists of three terms which correspond to the one overlapping and the two disjoint non-overlapping areas of the time intervals $[kT_s+\tau_l,\ (k+1)T_s+\tau_l]$ and $[kT_s+\tau_{l'},\ (k+1)T_s+\tau_{l'}]$. Each of the non-overlapping areas and the overlapping area (if any) in the corresponding integral of (9) result to a sum of n_u and n_c binary (±1) equiprobable i.i.d. R.V.'s respectively, where

$$\begin{aligned} 0 < n_u = min(N_s, |\delta_{l'} - \delta_l|) \le N_s \\ 0 \le n_c = N_s - n_u < N_s \end{aligned} \quad (11)$$

From (9), (10), and based on the above, the mutual interference between fingers l, l' can be written as:

$$Y_{l,l'}(k) = \frac{\sqrt{E_c}}{N_s}\alpha_l\alpha_{l'}cos(\theta_l - \theta_{l'})(2V^c_{l,l'}(k) + V^u_{l,l'}(k) + V^u_{l',l}(k)) \quad (12)$$

where $V^c_{l,l'}(k),\ V^u_{l,l'}(k),\ V^u_{l',l}(k)$ are independent R.V.'s. The R.V. $V^c_{l,l'}(k)$ is distributed according to $1 - 2X_c$ ($X_c \sim Binom(n_c, 1/2)$) and has zero mean and variance n_c. Each of $V^u_{l,l'}(k),\ V^u_{l',l}(k)$ is distributed according to $1 - 2X_u$ ($X_u \sim Binom(n_u, 1/2)$) and have zero mean and variance n_u. Conditioned on the path gains, each mutual interference term has zero mean and variance

$$var\{Y_{l,l'}(k)\} = \frac{2E_c}{N_s^2}\alpha_l^2\alpha_{l'}^2 cos^2(\theta_l - \theta_{l'})(2N_s - n_u) \quad (13)$$

Note that the mutual interference terms are in general correlated and the variance of $y_{IPI}(k)$ may be different than the sum of the variances (13). An important implication of (13) is that the power of the mutual interference between two fingers is increased with decreasing n_u, i.e. the performance deteriorates as the overlapping increases for path-delay differences smaller than T_s.

The decoder ignores the term corresponding to the IPI (9) and processes the observation (4) of coded symbol $d(k)$ as if it were only corrupted by the noise term (7). We define the average SNR at the input of the decoder:

$$\gamma_b = \frac{(\sum_{l=0}^{N_p-1} \mathcal{E}\{\alpha_l^2\})E_b}{N_0} \quad (14)$$

where $E_b = E_d/R$ is the energy per uncoded data symbol, and E_d is the energy per coded (transmitted) symbol, with $E_d = N_sE_c$. We can approximate the dependence of the power ratio between the IPI and the noise

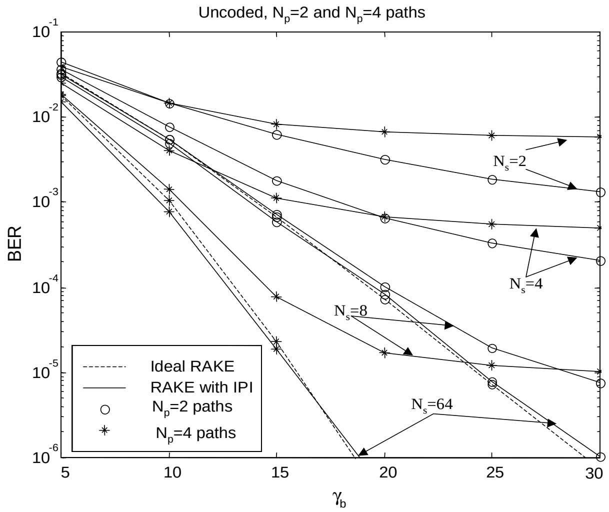

Figure 2: Effect of low SG on the performance of uncoded VSG-CDMA systems.

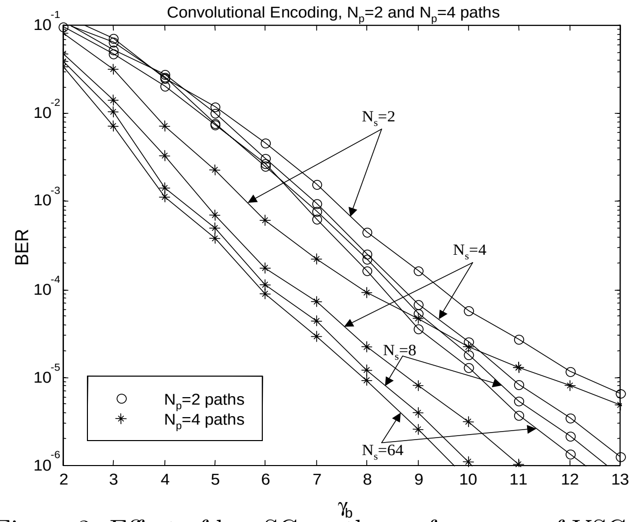

Figure 3: Effect of low SG on the performance of VSG-CDMA systems with Convolutional coding.

using (10), (13), and (8):

$$\frac{var\{y_{IPI}(k)\}}{var\{y_n(k)\}} \sim \frac{N_p^2 \gamma_b}{N_s} \qquad (15)$$

Some interesting observations can be made based on (15). First, we notice that for large SG N_s, the performance of the system approaches that of the "ideal RAKE", i.e. that of an N_p-th diversity combiner using MRC [2]. Second, we notice that the effect of IPI becomes more important when the system operates in the high SNR region. In systems where interference increases with increasing useful signal power, the performance typically reaches an error-floor. Finally, increasing the number of diversity branches N_p has conflicting effects on the power of IPI. Although it reduces IPI by reducing the required SNR for a desired performance (because of increased diversity order), at the same time it increases the number of interfering paths, thus increasing the power of IPI proportionally to N_p^2.

IV. PERFORMANCE RESULTS

In this section we present simulation results that demonstrate the effect of low SG on the performance of coded systems using Convolutional and Turbo codes. We consider two different channel profiles with $N_p = 2$ and $N_p = 4$ paths and the powers of all path gains are assumed time-invariant and equal. For convenience we normalize the channel coefficients in (14) such that $\sum_{l=0}^{N_p-1} \mathcal{E}\{\alpha_l^2\} = 1$. Because of (13) we assume that the delays $\delta_l = l, \ l = 0, ..., N_p - 1$ in order to assess the worst case performance. The fading rate is $\omega_D T_s = 0.03$ and we assume that perfect channel estimates are available to the decoders. We select the system parameters similar to the IS-2000 air-interface [12]. We consider wireless frames of $N_f = 384$ encoded symbols and overall coding rate R=1/2. For Convolutional encoding we use the IS-2000 channel encoder with R=1/2, L=9 (memory $\nu = 8$), and generators $f_0(D) = (753)_{oct}$, $f_1(D) = (561)_{oct}$ and we use the MLSE Viterbi algorithm for decoding. For Turbo encoding we use the IS-2000 code (initial rate R=1/3, punctured appropriately) composed of recursive systematic encoders with constraint length $L = 4$ (memory $\nu = 3$), and generators $[1, \ (1 + D + D^3)/(1 + D^2 + D^3)]$. The iterative decoder uses the Log-MAP algorithm and 10 iterations.

We expect from (15) that for a large SG the performance of the system with IPI will be close to the performance of the "ideal RAKE". Figure 2 presents the performance of an uncoded system and the corresponding performance of an MRC combiner with N_p degrees of diversity [2]. For $N_s = 64$ the performance of the system is approximately the same to that of the "ideal RAKE", therefore in the rest of this section we will use $N_s = 64$ as a benchmark of "IPI-free" performance. When the SG gets smaller and the SNR higher the effect of the IPI becomes dominant (15) and the system reaches an error floor. Figure 2 also shows that in the low SNR region where AWGN is dominant the performance with 4 paths is better than with 2 paths because of the higher degree of diversity. In the high SNR region where IPI is dominant the performance with 4 paths is worse than with 2 paths despite the increased diversity, because of the higher power of the self-interference (15).

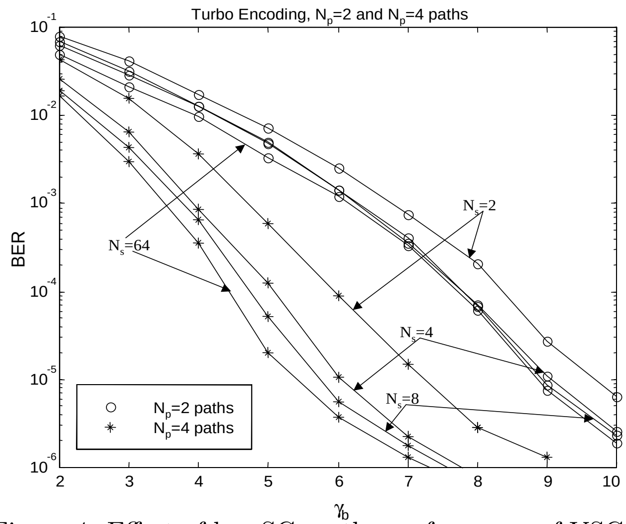

Figure 4: Effect of low SG on the performance of VSG-CDMA systems with Turbo coding.

The effect of low SG on the performance of coded systems with Convolutional and Turbo coding is depicted in Figures 3 and 4. Coding increases the degree of effective diversity of the system without increasing the power ratio of IPI over AWGN in (15). With Convolutional coding the desired performance ($BER \geq 10^{-6}$) is achieved in medium to low SNR region. The performance degradation is less than 1.2 dB for $N_s \geq 4$ and an error floor due to IPI is observed only for $N_s = 2$. With Turbo coding the desired performance is achieved for even lower SNR where IPI has a smaller impact. With $N_p = 4$ paths the performance degradation is less than 0.7 dB for $N_s \geq 4$ and 2 dB for $N_s = 2$. With $N_p = 2$ paths the degradation is negligible for $N_s \geq 4$ and 1 dB for $N_s = 2$. The flattening in performance is the same for all SG and is typical in Turbo codes due to their free distance properties and not the IPI.

V. CONCLUSIONS

In this paper we examined the effect of IPI in coded VSG-CDMA systems with low SG. A brief analysis of the self-interference helped us understand the degradation in performance due to the IPI compared to the "ideal RAKE" performance. In coded systems with Convolutional coding the degradation due to the IPI is moderate for SG as low as 4 and an error floor is observed only when the SG is 2. The impact of self-interference is even smaller with Turbo codes and a flattening in performance is due to the properties of the codes. The obtained results can be used as a benchmark to assess the trade-off between complexity increase and performance improvement achieved by SIC schemes.

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