On the Performance of Adaptive MMSE Detectors for a MC-CDMA System in Fast Fading Rayleigh Channels*

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ABSTRACT

Multi-Carrier CDMA (MC-CDMA) is a hybrid multiple access scheme combining Multi-Carrier Modulation (MCM or OFDM) with Direct Sequence Spread Spectrum (DS-SS). Although a common assumption in the analysis of the performance of the various MC-CDMA detectors is that they have perfect knowledge of the fading channel, this assumption is no longer valid as the fading rate (Doppler Spread) of the channel increases. In this paper we examine the performance of various forms of the Minimum Mean Square Error (MMSE) detector operating in a fast fading Rayleigh channel described by a Gauss-Markov state-space model. We show that, although adaptive detection based on the LMS and RLS algorithms is possible without explicit channel estimation, a low complexity detector, termed MMSE per Carrier, combined with an RLS channel estimator can achieve better performance as well as increased robustness to the selection of the adaptation parameters.

I. INTRODUCTION

There is currently an increasing research interest in exploiting the technology options, as well as seeking new discoveries, which would make possible the efficient deployment of Broadband Wireless Integrated Services Networks (B-WISN). One of the most challenging aspects in the design of such systems, which could determine their future success, is choosing a highly bandwidth efficient multiple access scheme, which at the same time demonstrates robustness against the hostile nature of the broadband radio channel. One such scheme is Multi-Carrier CDMA (MC-CDMA), which appears as a candidate for B-WISN systems.

From the several schemes combining Multi-Carrier Modulation with Direct Sequence Spread Spectrum (DS-SS) that have been proposed in the literature, the one proposed in [1] and others independently, has attracted the largest research interest so far. The basic idea of this scheme is to divide the available bandwidth into a large number of narrow subchannels, and spread each data symbol in the frequency domain by transmitting all the chips of a spread symbol at the same time, but in different orthogonal subchannels. Since the chips of all the symbols that form a multi-carrier block overlap in time, even high data rate information can be transmitted using a large MC symbol duration T, which drastically reduces ISI, allows for approximately flat fading in each subchannel, and combats the frequency selective fading of the channel by introducing a large degree of frequency diversity. The focus of this paper is on this MC-CDMA scheme.

Previous papers that investigated the performance of the MC-CDMA detectors (e.g. [2], [3]), made the assumption that the multiplicative complex channel coefficients, which describe the effect of the frequency selective channel, are perfectly known to the receiver. Recently the impact of channel estimation errors on the performance of MC-CDMA detectors attracted significant research interest, and different approaches were adopted: channel estimation by transmitting sounding MC-CDMA blocks consisting of trains of pulses was considered in [4]; pilot symbol aided channel estimation in the time and frequency dimensions was proposed in [5]; and decision directed channel estimation using Kalman filtering and assuming a known state-space model was analyzed in [6] for the Threshold Orthogonality Restoring Combining (TORC) detector. In this paper we follow the last approach, but we consider the more realistic case that the channel is described by a Gauss-Markov state-space model whose parameters are in general not known to the receiver. We examine the performance of two forms of the MMSE detector, which are termed in [1] as MMSE per User and MMSE per Carrier, and we consider adaptive schemes with explicit channel estimation using the LMS and RLS algorithms, which are compared to the optimum Kalman filter. Finally, we consider adaptive MMSE detectors using the LMS and RLS algorithms without explicit channel estimation, and we compare them to the forms employing channel estimation.

This paper is organized as follows: In Section II, a description of the channel model, of the MC-CDMA transmitter and receiver structure, and of the channel estimation process is given. In Section III, the various forms of the MMSE detector are described. In Section IV, simulation results on the system performance are presented, and in Section V, we give our conclusions and future directions.

*This work was supported in part by GTE Laboratories Inc., Waltham, MA.

II. SYSTEM DESCRIPTION

We consider a multiple access system where N_u users are transmitting simultaneously in a synchronous manner using Walsh-Hadamard orthogonal codes of length N_s. Therefore up to N_s users can transmit at the same time. The $n-th$ multicarrier block symbol (of duration T_b) for user i is formed by taking μ symbols $b_i^1(n),...,b_i^\mu(n)$ in parallel, spreading them with the user's spreading sequence $\mathbf{c}_i = [c_{1,i}...c_{Ns,i}]^T, c_{j,i} = \pm 1$, performing frequency interleaving, and placing the resulting $u_i^1(n),...u_i^{\mu Ns}(n)$ chips into the $N = \mu Ns$ available subchannels, each having width $\Delta f = 1/T_b$, by using an IFFT of size N. After performing a parallel to serial conversion, a guard interval is added, in the form of a cyclic prefix, and the signals of all the users are added and transmitted through the channel. The block diagram of the transmitter is depicted in Figure 1. In the rest of this paper, for simplicity of notation, we will concentrate only on one of the μ symbols each user transmits, by setting $\mu = 1$, and keeping in mind that the frequency interleaving function still exists. Also, we will consider binary symbols $b_k(n) = \pm 1, k = 1,...,N_u$ forming the data vector $\mathbf{b}(n) = [b_1(n),...,b_{Nu}(n)]^T$, where n is the time index denoting the $n-th$ symbol interval.

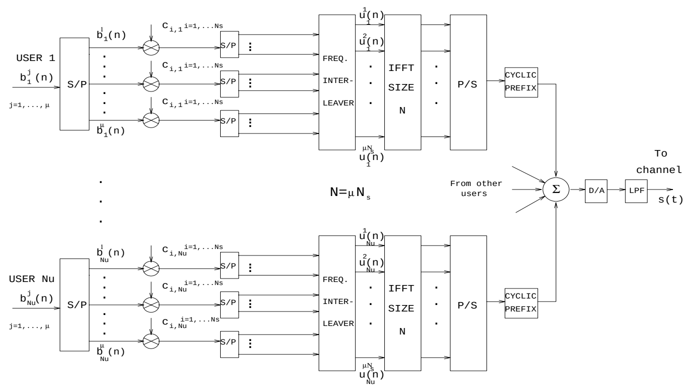

Figure 1: Transmitter block diagram.

The transmitted signal during the $n-th$ MC block symbol period can be written as follows:

$$s(t) = \sum_{k=1}^{N_u}\sum_{l=1}^{N_s}\sqrt{E_c}c_{l,k}b_k(n)e^{j\frac{2\pi l(t-T_G)}{T_b}} \quad (1)$$

where $t \in [nT,(n+1)T]$, $T = T_b + T_G$, T_G is the guard interval chosen to be at least equal to the delay time spread T_m of the channel, and E_c is the energy per chip.

We consider the case of a fast fading, frequency selective, Rayleigh channel, and we assume that the channel is not changing during one MC symbol interval. We allow, however, for variations during successive symbol intervals. Because of the large symbol duration, the fading in each subchannel is approximately flat, and is described by a multiplicative complex channel coefficient $h_l(n), l = 1,...,N_s$, which is a Gaussian distributed, complex, discrete time random process. Because of the frequency interleaving function, the channel complex coefficient processes will be considered independent. This is valid in the case when $\mu/T_b > 1/T_m$. We assume that each of these random processes is described by a first order, Gauss-Markov model, of the following form:

$$h_l(n+1) = fh_l(n) + \chi_l(n), \qquad l = 1,...,N_s \quad (2)$$

where $\chi_l(n)$ is a zero mean, white Gaussian noise process, with autocorrelation

$$E\{\chi_l(n)\chi_l^*(k)\} = 2\sigma_h^2\delta_{n,k} \quad (3)$$

and $\delta_{n,k}$ is the Kronecker delta. The parameter f corresponds to an exponentially decaying channel time correlation function, and is related to the coherence time $(\Delta t)_c$ and the $3-dB$ Doppler Spread bandwidth B_d as follows:

$$f = e^{-\omega_d T} \quad (4)$$

where $\omega_d = \pi B_d = 2\pi/(\Delta t)_c$.

Because of the existence of a guard interval with duration at least equal to the channel's delay spread, there is no intersymbol interference, and the signal received by user i can be described by the following equation:

$$r(t) = \sum_{k=1}^{N_u}\sum_{l=1}^{N_s}\sqrt{E_c}h_l^{(i)}(n)c_{l,k}b_k(n)e^{j\frac{2\pi l(t-T_G)}{T_b}} + n(t) \quad (5)$$

where $t \in [nT,(n+1)T]$, $h_l^{(i)}(n)$ are the complex channel coefficients which describe the channel between the transmitter and the user i, and $n(t)$ is the AWGN. For simplicity of notation in the rest of the paper, the superscript (i) will be dropped. At the receiver, the signal is sampled at a rate N/T_b, the samples which correspond to the cyclic prefix are discarded, an FFT of size N is performed, and frequency deinterleaving takes place. The vector $\mathbf{r}(n) = [r_1(n),...,r_{Ns}(n)]^T$ at the output of the deinterleaver is given in matrix notation by the following equation:

$$\mathbf{r}(n) = \sqrt{E_c}\mathbf{H}(n)\mathbf{C}\mathbf{b}(n) + \boldsymbol{\eta}(n) \quad (6)$$

where $\mathbf{H}(n) = diag\{h_1(n),...,h_{Ns}(n)\}$, matrix $\mathbf{C} = [\mathbf{c}_1 \mid ... \mid \mathbf{c}_{Nu}]$ is the N_sxN_u matrix whose columns are the spreading sequences of the users, $\mathbf{b}(n)$ is the data vector of the users, and $\boldsymbol{\eta}(n) = [\eta_1(n),...,\eta_{N_s}(n)]^T$ is a vector containing zero mean, uncorrelated complex Gaussian noise samples.

We distinguish two forms of decision directed adaptive detectors, depending on the channel estimation process. When the MMSE detector is implemented adaptively without explicit channel estimation, the observation vector $\mathbf{r}(n)$ is used by the MMSE detector in order to obtain an estimate of the current data vector $\hat{\mathbf{b}}(n)$, and both $\mathbf{r}(n)$ and $\hat{\mathbf{b}}(n)$ are used to update the detector coefficients for the next symbol interval. On the other hand, when the detector uses explicit channel estimates the observation vector $\mathbf{r}(n)$ is fed to the MMSE detector and to the channel estimator, which in addition uses estimates of previous data vectors from the output of the detector. The block diagram of the receiver is depicted in Figure 2.

In this paper we consider three ways of obtaining the channel estimates $\hat{h}_1(n),...,\hat{h}_{Ns}(n)$. If the process model (2) is known to the receiver, the Kalman filter gives the best estimates in the MMSE sense. In the general case, when the process

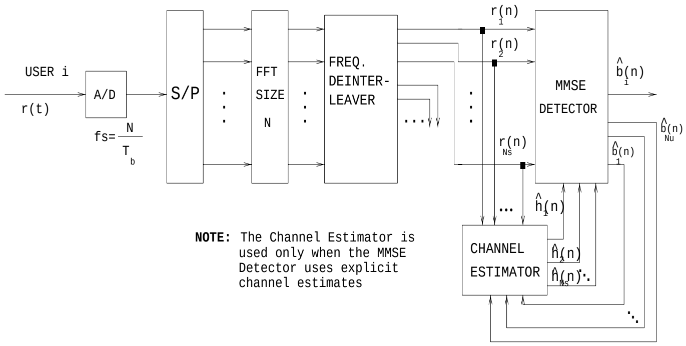

Figure 2: Receiver block diagram.

model is not known, the estimate of the channel coefficients can be obtained by using the LMS and the RLS algorithms. In all three cases, the estimator's input is the vector $\mathbf{r}(n)$, which represents the measured quantity, and the estimate of the data vector $\hat{\mathbf{b}}(n)$. The equation that describes the measurement vector is derived from (6) by rearranging its terms:

$$r_l(n) = d_l(n)h_l(n) + \eta_l(n), \qquad l = 1, ..., N_s \tag{7}$$

where

$$d_l(n) = \sqrt{E_c}\sum_{k=1}^{N_u} c_{l,k} b_k(n) \tag{8}$$

and $\eta_l(n)$ is the measurement noise process, which is white Gaussian, with zero mean and variance $2\sigma^2$. The algorithms of the three estimators are described as follows [7], [8]:

1. **Kalman Filter:** The new channel estimate at time $n+1$ using information available up to time n is

$$\hat{h}_l(n+1) = [f - K_l(n)\hat{d}_l(n)]\hat{h}_l(n) + K_l(n)r_l(n) \tag{9}$$

   where the term $K_l(n), l = 1, ..., N_s$ is the Kalman gain, which is related to the error variance $E_l(n) = \mathcal{E}\{|h_l(n) - \hat{h}_l(n)|^2\}$ by the following equation:

$$K_l(n) = \frac{fE_l(n)\hat{d}_l(n)}{\hat{d}_l^2(n)E_l(n) + 2\sigma^2} \tag{10}$$

   The estimation error variance is calculated in an iterative way:

$$E_l(n|n-1) = f^2 \frac{2\sigma^2 E_l(n-1|n-2)}{\hat{d}_l^2(n-1)E_l(n-1|n-2) + 2\sigma^2} + 2\sigma_h^2 \tag{11}$$

2. **LMS Estimator:** The new channel estimate at time $n+1$ using information available up to time n is

$$\hat{h}_l(n+1) = \hat{h}_l(n) + \mu[r_l(n) - \hat{h}_l(n)\hat{d}_l(n)]\hat{d}_l^*(n) \tag{12}$$

   where $l = 1, ..., N_s$, and $\mu > 0$ is the step size of the LMS algorithm.

3. **RLS Estimator:** The new channel estimate at time $n+1$ using information available up to time n is

$$\hat{h}_l(n+1) = \hat{h}_l(n) + [r_l(n) - \hat{h}_l(n)\hat{d}_l(n)]K_l^*(n) \tag{13}$$

   where the term $K_l(n)$ is the Kalman gain given by

$$K_l(n) = \frac{P_l(n)\hat{d}_l(n)}{\lambda + P_l(n)\hat{d}_l^2(n)} \tag{14}$$

   $P_l(n)$ is given by the following recursion

$$P_l(n+1) = \lambda^{-1}(1 - K_l(n)\hat{d}_l^*(n))P_l(n) \tag{15}$$

   and $0 < \lambda < 1$ is the forgetting factor of the RLS algorithm.

In the above equations $\hat{d}_l(n)$ denotes the quantity given in (8), as this is obtained by using the estimate of the data vector $\hat{\mathbf{b}}(n)$.

III. THE MMSE DETECTOR

The MMSE detector for the MC-CDMA system under consideration has been examined under the assumption of perfectly known channel in [1] and [2], while its performance using pilot symbol aided channel estimation has been evaluated in [5]. Two different forms of this detector are distinguished, although for the MC-CDMA system under consideration the two detectors become the same when $N_u = N_s$:

1. **MMSE per User Detector:** The optimization criterion is to find a matrix $\mathbf{W_0}(n)$ such that

$$\mathbf{W_0}(n) = \underset{\mathbf{W}(n)}{argmin}\ \mathcal{E}\{\|\mathbf{b}(n) - \mathbf{W}(n)\mathbf{r}(n)\|^2\} \tag{16}$$

   Then, the data vector estimate is obtained as follows:

$$\hat{\mathbf{b}}(n) = sgn[\mathcal{R}e\{\mathbf{W_0}(n)\mathbf{r}(n)\}] \tag{17}$$

   The solution of (16) is obtained by applying the orthogonality principle and has the form:

$$\mathbf{W_0}(n) = \mathbf{R_{br}}(n)\mathbf{R_{rr}^{-1}}(n) \tag{18}$$

   where

$$\mathbf{R_{br}}(n) = \mathcal{E}\{\mathbf{b}(n)\mathbf{r}^H(n)\} = \sqrt{E_c}\mathbf{C}^T\mathbf{H}^*(n)$$
$$\mathbf{R_{rr}}(n) = \mathcal{E}\{\mathbf{r}(n)\mathbf{r}^H(n)\} = E_c\mathbf{HCC}^T\mathbf{H}^*(n) + 2\sigma^2\mathbf{I}$$

2. **MMSE per Carrier Detector:** The optimization criterion is to find a matrix $\mathbf{W_0}(n) = diag\{W_{0,1}(n), ..., W_{0,N_s}(n)\}$ such that

$$W_{0,i}(n) = \underset{W_i(n)}{argmin}\ |\ [\mathbf{Cb}(n)]_i - W_i(n)[\mathbf{r}(n)]_i\ |^2 \tag{19}$$

where $[\cdot]_i$ denotes the ith element of the vector inside the brackets. The solution to (19) is the following

$$W_{0,i}(n) = \frac{h_i^*(n)}{|h_i(n)|^2 + \frac{2\sigma^2}{N_u E_c}}, \qquad i = 1, ..., N_s \tag{20}$$

and the data vector estimate is given by

$$\hat{\mathbf{b}}(n) = sgn[\mathcal{R}e\{\mathbf{C}^T \mathbf{W_0}(n)\mathbf{r}(n)\}] \tag{21}$$

We consider two forms of adaptive implementation of the MMSE detectors. According to the first, the channel coefficients $h_l(n)$ are explicitly estimated using one of the methods described in the previous section, and then the matrix $\mathbf{W_0}(n)$ is calculated by using the channel estimates $\hat{h}_l(n)$ instead of $h_l(n)$ in (18) and (20). According to the second, (16) and (19) are solved in an adaptive form using the LMS and RLS algorithms. In this case, the adaptive MMSE per User algorithm takes the following form:

1. **LMS Algorithm:**

$$\mathbf{W}(n+1) = \mathbf{W}(n) + \mu[\mathbf{b}(n) - \mathbf{W}(n)\mathbf{r}(n)]\mathbf{r}^H(n) \tag{22}$$

2. **RLS Algorithm:**

$$\mathbf{W}(n+1) = \mathbf{W}(n) + [\mathbf{b}(n) - \mathbf{W}(n)\mathbf{r}(n)]\mathbf{k}^H(n) \tag{23}$$

$$\mathbf{k}(n) = \frac{\mathbf{P}(n)\mathbf{r}(n)}{\lambda + \mathbf{r}^H(n)\mathbf{P}(n)\mathbf{r}(n)} \tag{24}$$

$$\mathbf{P}(n+1) = \lambda^{-1}(\mathbf{I} - \mathbf{k}(n)\mathbf{r}^H(n))\mathbf{P}(n) \tag{25}$$

Similar expressions can be derived for the MMSE per Carrier detector. The adaptive MMSE per User detector (22) using the LMS algorithm has been suggested in [1] and [9], although no performance results were given. In the rest of this paper we will focus on the performance of the MMSE per Carrier detector with explicit adaptive channel estimation, and on the adaptive MMSE per User detector described in (22) and (23).

IV. PERFORMANCE EVALUATION

In this section we present several examples that illustrate the performance of the MC-CDMA system under consideration. We assume that correct decisions are fed back, and whenever SNR is mentioned it refers to the average SNR per bit γ_b, which is defined as follows:

$$\gamma_b = \frac{N_s E_c \mathcal{E}\{|h_l(n)|^2\}}{2\sigma^2} = \frac{N_s E_c \sigma_h^2}{(1-f^2)\sigma^2} \tag{26}$$

In all cases we normalize the channel coefficients so that

$$\mathcal{E}\{|h_l(n)|^2\} = 1 \;\Leftrightarrow\; \frac{2\sigma_h^2}{1-f^2} = 1 \tag{27}$$

From (27) it is obvious that, in order to keep a constant reference in the definition (26) of SNR, the variance σ_h^2 should be varied accordingly as f changes for different fading rates $\omega_d T$.

When LMS and RLS based channel estimators are used, the selection of the step size μ and the forgetting factor λ depends on the channel fading rate and the SNR. For each fading rate there is an optimum selection of the adaptation parameters, as this is depicted in Figures 3 and 4. We notice that the performance of the MMSE detector with LMS channel estimation is much more sensitive to the selection of the step size μ, than the performance of the RLS-based detector to the variations of the forgetting factor λ.

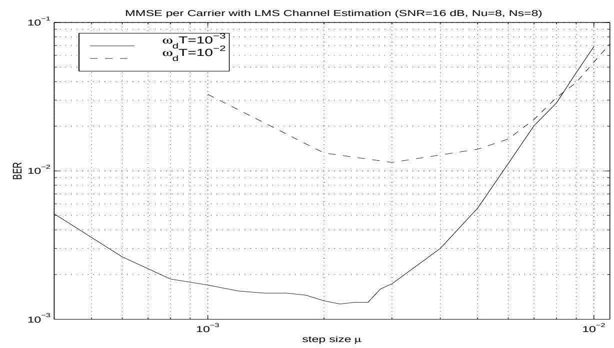

Figure 3: BER as a function of the step size μ.

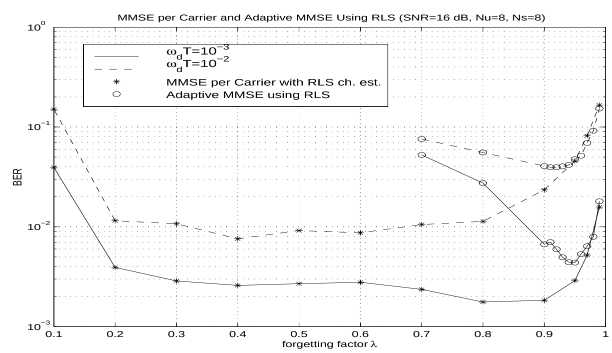

Figure 4: BER as a function of the forgetting factor λ.

Since the Kalman filter gives the optimum estimates in the MMSE sense, it is expected that the BER of the MMSE detector using the Kalman filter will be a lower bound on the probability of error when other channel estimators, such as the LMS and RLS, are used. This is shown in Figures 5 and 6, where the BER of the MMSE per Carrier detector with LMS and RLS channel estimation is plotted as a function of SNR. It can be seen that the performance of the detector, when RLS channel estimation is used, is much closer to that of the Kalman filter than when LMS estimation is used, at the expense of increased complexity.

As described in the previous section, the MMSE detector can be implemented adaptively without explicit channel estimation. Although it has been proposed in the literature, computer experiments showed that the LMS adaptive detector described in (22) demonstrates very slow convergence capabilities, and it is unable to perform well ($BER < 10^{-1}$) even for slow fading channels ($\omega_d T = 10^{-4}$), for any value of the

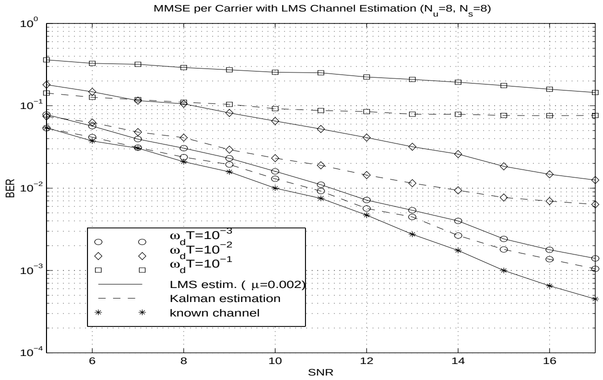

Figure 5: BER as a function of γ_b $(10^{-3} \leq \omega_d T \leq 10^{-1})$.

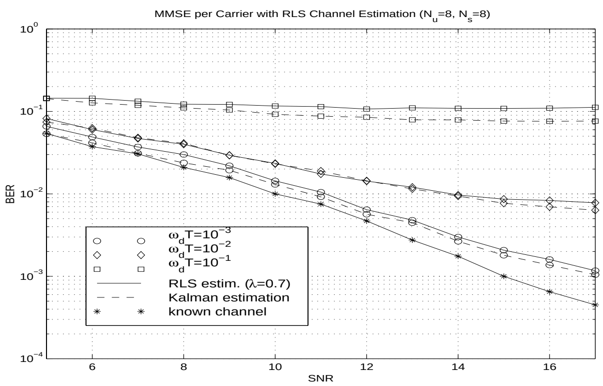

Figure 6: BER as a function of γ_b $(10^{-3} \leq \omega_d T \leq 10^{-1})$.

step size μ. The RLS adaptive MMSE detector (23), on the other hand, manages to track the channel variations up to a certain point, although it is more sensitive to the selection of λ, as it can be seen in Figure 4, and performs worse than the detector employing explicit channel estimation, as it is shown in Figure 7.

V. CONCLUSIONS

In this paper the performance of different adaptive forms of the MMSE detector of a MC-CDMA system operating in a fast fading, multipath Rayleigh channel, was investigated. The MMSE per Carrier detector with RLS channel estimation was found to be robust with respect to adaptation parameter variations, and approached the performance of the detector using optimal Kalman filtering. The adaptive detector without channel estimation using the LMS algorithm was unable to follow the channel variations and demonstrated poor performance. The more complex detector using RLS adaptation performed better, but did not achieve very good performance either.

The MMSE detector per Carrier employing RLS channel estimation combines a low complexity structure, robustness in parameter variations, and very good performance. We are currently investigating the robustness of the detector with respect to error propagation, when the estimated symbols are fed back to the channel estimator.

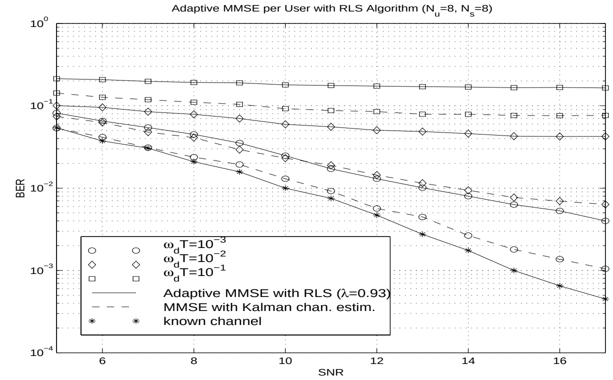

Figure 7: BER as a function of γ_b $(10^{-3} \leq \omega_d T \leq 10^{-1})$.

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