Performance of Multi-Code CDMA Systems with Concatenated Coding and Iterative Decoding in Multipath Rayleigh Fading Channels*

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ABSTRACT

Multi-Code CDMA (MC-CDMA) is proposed as an alternative multiple access scheme which provides the flexibility, high data rates, and bandwidth on demand features necessary in next generation wireless integrated services networks. In a conventional MC-CDMA configuration (e.g in IS-95B) a user can transmit information in multiples (M times) of a basic data rate by using M orthogonal supplemental DS-CDMA channels. In this paper, motivated by the remarkable performance of Turbo codes with large interleaving lengths in AWGN channels, we investigate the performance of a MC-CDMA system employing different concatenated encoding schemes combined with iterative decoding. Furthermore, we examine the Frame Error Rate performance of the various schemes by considering CRC codewords in each of the M frames corresponding to each data block. We show that coding and interleaving at the block level improves the Block Error Rate (BlER) performance, especially when Turbo schemes are used, while the extra CRC overhead is beneficial mainly when convolutional codes are used.

I. INTRODUCTION

Multi-Code CDMA (MC-CDMA) [1] is a flexible multiple access scheme that allows for multi-rate data transmission using Direct Sequence Spread Spectrum (DS-SS) with constant processing gain. According to this scheme, each user can transmit data at multiples M of a basic rate by dividing each data block into M parallel streams of data, and using orthogonal spreading codes concatenated with long PN sequences to spread each stream as in DS-SS. MC-CDMA has already been applied in IS-95B [2] and is considered as a candidate for next generation CDMA systems because it combines backwards compatibility with existing systems, flexibility in transmission rates, and bandwidth on demand features.

In MC-CDMA systems an M-fold increase in data rate is accompanied by a proportional M-fold increase in user transmission power. Any technique that could exploit the MC-CDMA structure in order to improve the required SNR for a given QoS requirement without modifying the fundamental air-interface characteristics, would be particularly beneficial to the CDMA system capacity. One such method could be using Turbo codes [3],[4], which were recently applied to CDMA systems producing promising results [5],[6]. The inherent characteristic of MC-CDMA having longer data blocks divided in parallel frames is exploited by concatenated encoding schemes with iterative decoders [7], since it is known that these schemes benefit from larger interleaving lengths.

In this paper we consider the generalized configuration of a MC-CDMA system employing a serially concatenated scheme with an outer encoder having as input the long data block before serial to parallel (S/P) conversion takes place, and M inner encoders, one for each parallel spreading code branch. The outer and inner encoders use convolutional, parallel, or serial Turbo codes. We examine the performance of several special cases of this generalized configuration. First, we examine the case with no outer encoder and M inner encoders, which is termed "post-S/P" scheme (IS-95B corresponds to this configuration with convolutional inner encoders). Second, we examine the case with an outer encoder and no inner encoders which is termed "pre-S/P" scheme. Finally, covering the area between the two previous cases, we consider a serial scheme with outer

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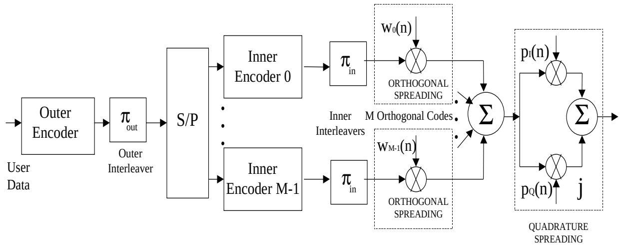

Figure 1: Generalized MC-CDMA transmitter with concatenated encoding.

and inner convolutional encoders (forming a type of serial Turbo) which is termed "MC-Serial" scheme. We compare the performance of the above schemes in terms of Bit (BER) and Block (BlER) Error Rate, and we comment on the benefits of the system tracking the error statistics at the frame (after the long data block is serial to parallel divided) level through the use of CRC codewords.

The rest of this paper is organized as follows: in Section II we describe the generalized MC-CDMA system, as well as the coding and decoding schemes under consideration. In Section III we present and compare performance results of the selected special cases. Finally, in Section IV we present the conclusions of this investigation.

II. SYSTEM DESCRIPTION

A. Generalized MC-CDMA System

We consider the forward link of a Multi-Code CDMA system which is a generalized form of IS-95B [2]. Each user is transmitting in multiples (M times) of the basic data rate using M orthogonal Walsh-Hadamard codes of length N_s. In the generalized MC-CDMA configuration a long block of N_b source binary data symbols $b(k) \in \{\pm 1\}, k = 0, ..., N_b - 1$ is passed through the outer channel encoder of rate R_1. The encoded symbols are interleaved by the outer interleaver π_{out} of length N_b/R_1, and then placed into M parallel frames (S/P conversion). These M frames are encoded in parallel by the inner encoders of rate R_2 and the resulting symbols of each frame $d_i(k), k = 0, ..., N_f - 1, i = 0, ..., M - 1$ where $N_f = N_b/(MR)$ and $R = R_1 R_2$ is the overall code rate, are interleaved by each of the inner interleavers π_{in} of length N_f. The symbols of each frame are then orthogonally spread by using one of the N_s Walsh-Hadamard sequences $w_i(n), n = 0, ..., N_s - 1$. Ignoring the user's PN signature sequence, the chips of all frames $i = 0, ..., M - 1$ are added, and the resulting signal is quadrature spread by using two long PN sequences $p_I(n), p_Q(n)$. The resulting baseband complex signal $s(t)$ during a wireless frame period has the following form:

$$s(t) = \sum_{i=0}^{M-1} \sum_{k=0}^{N_f-1} \sum_{n=0}^{N_s-1} \sqrt{E_c/2} d_i(k) w_i(n)$$
$$(p_I(kN_s+n) + jp_Q(kN_s+n)) R_C(t - kT_s - nT_c) \tag{1}$$

where T_c and $T_s = N_s T_c$ are the chip and symbol durations respectively, $R_C(t)$ is a rectangular pulse of duration T_c, and E_c is the energy per transmitted chip. The block diagram of the generalized transmitter is depicted in Figure 1.

The wireless channel is assumed multipath fading channel with N_p paths modeled by independent zero mean complex Gaussian processes $a_l(t) = \alpha_l(t) e^{j\phi_l(t)}$ $l = 0, ..., N_p - 1$ modeled using Jakes' model [8]. The fading rate of the channel is defined as the product $\omega_D T_s$, where ω_D is the maximum Doppler shift which depends on the system operating frequency and the velocity of the user. Neglecting the effect of the long term shadowing and path loss, the received signal at the input of the receiver has the following form:

$$r(t) = \sum_{l=0}^{N_p-1} a_l(t) s(t - \tau_l) + n(t) \tag{2}$$

where $\tau_l, l = 0, ..., N_p - 1$ are the path delays and $n(t)$ is zero mean complex AWGN with power spectral density N_0.

At the receiver the signal is first sampled at the chip interval and the samples are fed into a RAKE receiver with L_p fingers. In each finger, the samples are multiplied by the complex conjugate of the corresponding channel coefficients (MRC combining), and then the real and imaginary parts are correlated with appropriately delayed versions of PN sequences $p_I(n)$ and $p_Q(n)$ respectively. The output of each finger is formed by adding the outputs of the I and Q correlators, and the output of the RAKE receiver by adding the outputs of all fingers. The output of the RAKE receiver is fed into M parallel branches, where the observations of the M frames are formed after orthogonal despreading with the corresponding Walsh sequences and deinterleaving. The symbol observations of the i-th frame $y_i(k), k = 0, ..., N_f - 1$ at the input of the i-th inner decoder have the following form:

$$y_i(k) = (\sum_{l=0}^{L_p-1} \alpha_l^2(k)) \sqrt{2E_c} d_i(k) + 1/N_s \sum_{l=0}^{L_p-1} \alpha_l(k)$$
$$(\sum_{n=0}^{N_s} (p_I(kN_s + n - \tau_l) w_i(n) \eta_I(n) +$$
$$p_Q(kN_s + n - \tau_l) w_i(n) \eta_Q(n))) + ISI + MCI \tag{3}$$

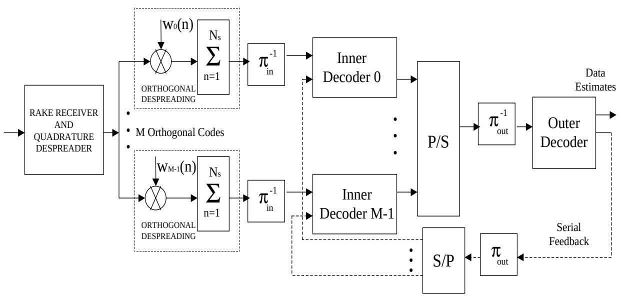

Figure 2: Generalized MC-CDMA receiver with iterative decoding.

where $\eta_I(n), \eta_Q(n)$ are the real and imaginary parts of the AWGN noise samples (zero mean Gaussian with variance $N_0/2$), and ISI and MCI are the residual Intersymbol and Multi-Code Interference respectively caused by imperfect PN sequence autocorellation properties.[1] In an ideal system the noise terms contributed by each finger in (3) would be independent and there would be no ISI and MCI terms. Note that in (3), as well as in the rest of this paper, we assume that the channel coefficients are constant during one symbol duration T_s, although they may change over successive symbol intervals.

The main component of all the iterative decoders is the Maximum A-Posteriori Probability (MAP) module, which is based on the Log-MAP algorithm [9], a variation of the original BCJR algorithm. Each MAP module accepts observations of the coded data and a-priori information about the uncoded data as inputs and produces Log-Likelihood Ratios (LLR's) and "extrinsic" information about the coded or uncoded data as outputs. The term extrinsic information denotes the incremental LLR information contributed by the MAP decoder.

The inner decoders accept as inputs the channel observations and a-priori information from the outer decoder (serial feedback), and produce extrinsic information, which is placed into one large block (P/S conversion), deinterleaved, and then fed as observation of the outer decoder. The outer decoder receives no a-priori information. It produces LLR's which are used to estimate the data sequence, and extrinsic information which is passed as a-priori information to the inner decoders. The block diagram of the generalized receiver is depicted in Figure 2.

B. Special Cases

In the rest of this paper we examine the performance of several special cases of the generalized MC-CDMA system. First we consider the case with no outer encoder and M inner encoders, which is termed "post-S/P" scheme. The inner decoders provide hard estimates for the uncoded data contained in each of the M frames. A convolutional, a parallel, and a serial Turbo codes are selected as inner encoders. For the convolutional code a Viterbi algorithm is used for MLSE decoding, while for the parallel and the serial Turbo codes MAP modules are used for iterative decoding.

The second special case includes an outer encoder and no inner encoders, and is termed "pre-S/P" scheme. The encoding and decoding is performed at the block level, and the interleavers involved are M times longer than the corresponding ones in the "post-S/P" case. Again, a convolutional, a parallel, and a serial Turbo codes are selected as outer codes, and decoding is performed similarly to the "post-S/P" case.

Finally, we consider a serial scheme with outer and inner convolutional encoders which is termed "MC-Serial" scheme. This is essentially a serial Turbo scheme with the outer encoding performed at the block level and the inner at the frame level. The iterative decoding is performed as described for the generalized receiver, and involves MAP modules for the inner and the outer decoders.

III. PERFORMANCE RESULTS

In this section we present simulation results on the performance of the different MC-CDMA configurations. In all cases we assume a channel with three independently fading paths with power ratios 2 and 4 dB and arrival delays $5T_c$ and $10T_c$ between the first and second, and first and third paths respectively. The channel fading rate is assumed $\omega_D T_s = 0.027$. Furthermore, we assume that perfect channel estimates are available and that the RAKE receiver has three fingers that track continuously the three signal arrivals ($L_p = N_p$). The definition of average SNR per data binary symbol γ_b is derived from (3) by neglecting the correlation between the thermal noise contributions of the three RAKE fingers and the ISI/MCI terms, although they are still present in the implemented system:

$$\gamma_b = \frac{\mathcal{E}\left(\sum_{l=0}^{L_p-1} \alpha_l^2\right) E_b}{N_0} \tag{4}$$

where $E_b = E_d/R$ is the energy per uncoded data symbol, and E_d is the energy per coded (transmitted) symbol, which is related to the energy per chip E_c according to $E_d = N_s E_c$. In the

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[1] The effect of possible Multi-User interference is not considered.

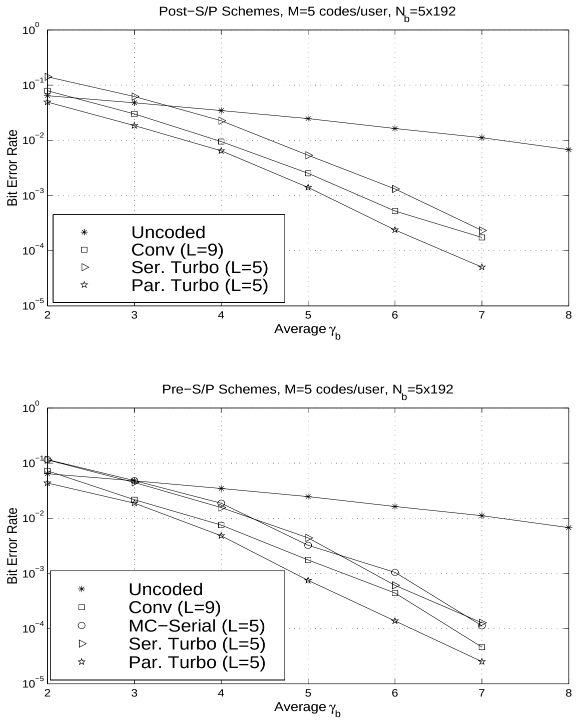

Figure 3: Bit Error Rate performance comparisons with M=5 codes/user.

results that follow we normalize the channel coefficients such that $\mathcal{E}\big(\sum_{l=0}^{L_p-1}\alpha_l^2\big)=1$.

Following the IS-95B [2] air interface we consider wireless frames of encoded length 384 symbols. In all encoding schemes overall code rate R=1/2 is used, and therefore the uncoded block is $N_b = 192 \times M$ symbols long. For convolutional encoding we use the IS-95B channel encoder (R=1/2, L=9, generators $f_0(D) = (753)_{oct}$, $f_1(D) = (561)_{oct}$). For parallel Turbo encoding we use a code (initial rate R=1/3, punctured appropriately) composed of recursive systematic encoders with constraint length L=5 ([1, $(1+D^2+D^3+D^4)/(1+D+D^4)$]). Finally, for serial Turbo codes (MC-Serial scheme included) the above recursive systematic codes are used as inner component codes, and an R=1/2 convolutional code with L=5 ($[1+D^3+D^4, 1+D+D^2+D^4]$) as outer. These codes are punctured to R_{out}=3/4 and R_{in}=2/3 for overall rate R=1/2. In all cases we use a maximum number of 10 iterations for iterative decoding.

The Bit Error Rate (BER) performance of the post-S/P and pre-S/P[2] schemes is depicted in Figure 3. We notice that the BER performance of the pre-S/P schemes is better than the post-S/P for all encoding schemes, although the improvement caused by larger interleaving lengths is relative small. The schemes using parallel Turbo codes display the best performance.

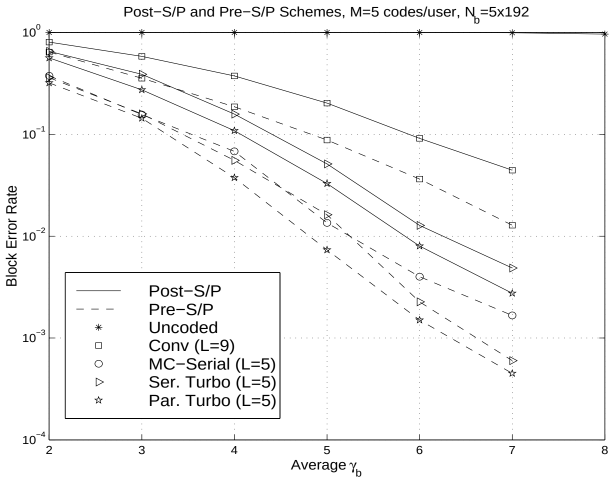

Figure 4: Block Error Rate performance comparisons with M=5 codes/user.

Comparing the performance between the post-S/P and the pre-S/P schemes reveals a different situation when Block Error Rate (BlER) is the chosen figure of merit. This is depicted in Figure 4. In this case we notice that the performance of the pre-S/P schemes is much better, with all systems benefiting by the longer interleaving length of the outer interleaver π_{out}. The improvement in high SNR's is even larger for the Turbo schemes, which seem to benefit from the longer interleavers between their component codes. It has to be noted however, that since groups of M encoded symbols of each block are subject to the same fading/noise statistics, the gain by increasing the Turbo interleaver lengths by M times is smaller than if all encoded symbols of each block had been transmitted in M successive wireless frames [6]. Another interesting remark drawn from Figure 4 is that the BlER performance of the MC-Serial scheme is between the pre-S/P and the post-S/P schemes, with its slope similar to the post-S/P serial Turbo scheme.

From Figures 3 and 4 we notice that the pre-S/P schemes demonstrate a burstier error behavior when compared to the post-S/P schemes: much fewer blocks contain a larger number of errors. The same trend is observed when comparing the convolutional with the parallel and serial Turbo schemes. This observation motivated us to examine the error statistics at the frame level. In a practical system this would mean that a CRC codeword would be inserted in all M frames corresponding to each user block. The Frame Error Rate (FER) performance of the different schemes is shown in Figure 5. We observe that, as ex-

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[2] Although the MC-Serial scheme incudes both post-S/P and pre-S/P encoders, for illustrative purposes in all figures we show it as part of the pre-S/P schemes.

pected, all post-S/P, the MC-Serial, and the pre-S/P convolutional schemes benefit the most when FER is the chosen figure of merit. The BlER and FER performance of both the pre-S/P parallel and serial Turbo schemes is practically the same, and remains the best among all schemes.

IV. CONCLUSIONS

In this paper we examined the performance of several special cases of a generalized MC-CDMA system employing concatenated coding and iterative decoding. We compared the performance of convolutional, parallel, and serial Turbo schemes used as inner or outer decoders. We showed that there is a large Block Error Rate performance gain when the encoding takes place before the Serial-to-Parallel conversion inherent to MC-CDMA systems. For high SNR's the performance gain caused by increased interleaving lengths is larger for schemes using parallel and serial Turbo schemes.

We also examined the effect of tracking the error statistics at the frame, instead of the block level. We concluded that mainly the post-S/P and all convolutional schemes benefit when comparing BlER and FER. Because of the extra overhead caused by CRC's at the frame level, we believe that it should be considered only when convolutional codes are used, since this is the only way for a convolutional scheme to approach the superior FER performance of Turbo schemes.

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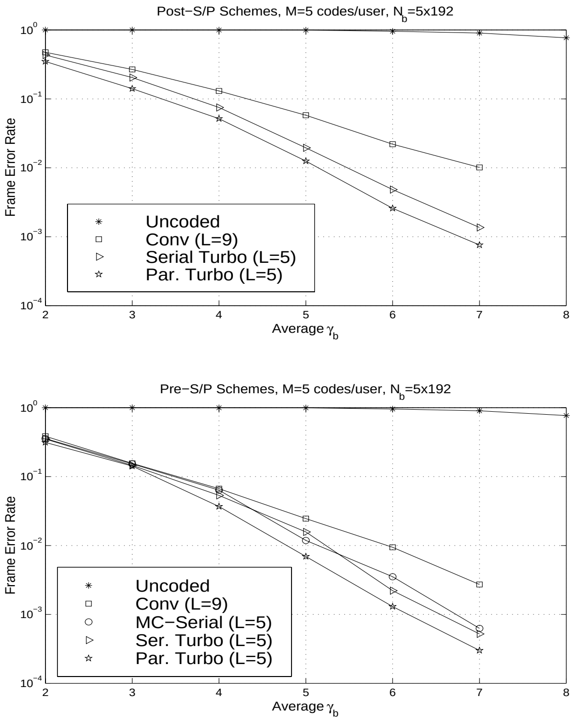

Figure 5: Frame Error Rate performance comparisons with M=5 codes/user.

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