Performance Comparisons of Concatenated Codes with Iterative Decoding for DS-CDMA Systems with Application to IS-95-based Cellular Systems[*]

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ABSTRACT

The remarkable performance of Turbo codes for low SNR's in AWGN channels has encouraged the application of Turbo codes to systems operating in different kind of channels. In this paper we consider an IS-95-based CDMA system operating in a multipath Rayleigh fading channel. We replace the convolutional encoder by a parallel and a serial Turbo encoding scheme, and by a serial concatenation of an outer convolutional and an inner parallel Turbo code, and instead of the Viterbi decoder we use iterative decoding based on the MAP algorithm. We perform BER and FER comparisons by varying system parameters such as the constraint length of the component convolutional codes, the interleaver lengths, and the wireless channel fading rate. The results obtained show that the concatenated schemes outperform the more complex convolutional scheme even for relatively short interleaving lengths, especially when FER is the desired figure of merit.

I. INTRODUCTION

The near optimum performance of Parallel Concatenated Recursive Systematic Convolutional codes (parallel Turbo codes) in AWGN channels [1] has prompted an intense research activity on the area of concatenated coding schemes employing interleavers. It has also made the application of Turbo codes very promising in a variety of areas, such as magnetic recording, satellite, and terrestrial cellular communications systems.

One application area which has attracted particularly active research is cellular communications systems, including both TDMA and CDMA. Examples of IS-95-based CDMA systems employing Turbo codes and/or iterative decoding can be found in [2], [3]. The emergence of next generation integrated services systems, such as those specified in the IMT-2000 initiative, offering services which can tolerate relatively long delays makes concatenated encoding with iterative decoding a strong candidate. Recently, application of parallel Turbo codes in the context of both the W-CDMA [4] and cdma2000 RTT proposals have produced promising results.

In this paper we investigate various design alternatives in applying concatenated codes employing interleavers and iterative decoding to a DS-CDMA system with coherent detection. The system under consideration is based on the forward link IS-95 standard and its next generation extensions and operates in a multipath Rayleigh fading channel. We examine three different encoding schemes: a "parallel Turbo" scheme [1] with two component convolutional codes in parallel, a "serial Turbo" scheme [5] with two component convolutional codes in series, and a serially concatenated scheme (termed "hybrid Turbo") with a convolutional outer code and a parallel Turbo inner code, similar to the one proposed in [6] where the outer code was a block BCH code. We compare their performance to that of the IS-95 convolutional encoder for overall code rates R=1/2 (rate I option) and R=3/4 (rate II option), and for different constraint lengths of the component codes. We also investigate the effect of varying the number of wireless frames containing one interleaved data block, and the channel fading rate.

The rest of this paper is organized as follows: in Section II we describe the link level of the DS-CDMA system, as well as the coding and decoding schemes under consideration. In Section III we present and compare performance results of the different decoders. Finally, in Section IV we present the conclusions of this investigation.

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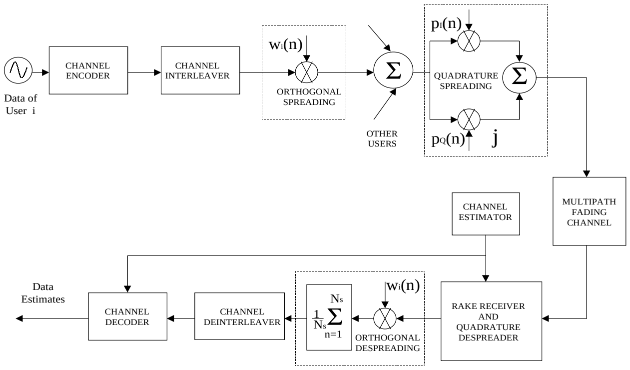

Figure 1: Baseband CDMA block diagram.

II. SYSTEM DESCRIPTION

A. Link Level of the DS-CDMA System

We consider a system which corresponds to the forward link of an IS-95-based [7] DS-CDMA system where N_u users are transmitting in a synchronous manner using orthogonal Walsh-Hadamard codes of length $N_s = 64$. A block of N source binary data symbols $b_i(k) \in \{\pm 1\}, k = 0, ..., N-1$ of user i are passed through the channel encoder and the encoded binary symbols $d_i(k), k = 0, ..., N_f - 1$ are placed into frames of length N_f, where $N_f = N/R$ and R is the overall code rate. These frames are then passed through a channel block interleaver of the same length, and the symbols are orthogonally spread by using one of the 64 Walsh-Hadamard sequences $w_i(n), n = 0, ..., N_s - 1$. Ignoring the user's PN signature sequence, the chips of all users $i = 0, ..., N_u - 1$ are added, and the resulting signal is quadrature spread by using two long PN sequences $p_I(n), p_Q(n)$. The resulting baseband complex signal $s(t)$ during a frame period has the following form:

$$\begin{aligned} s(t) = & \sum_{i=0}^{N_u-1} \sum_{k=0}^{N_f-1} \sum_{n=0}^{N_s-1} \sqrt{E_c/2}\, d_i(k) w_i(n) \\ & (p_I(kN_s + n) + jp_Q(kN_s + n)) R_C(t - kT_s - nT_c) \end{aligned} \tag{1}$$

where T_c and $T_s = N_s T_c$ are the chip and symbol durations respectively, $R_C(t)$ is a rectangular pulse of duration T_c, and E_c is the energy per transmitted chip. The block diagram of the transmitter is depicted in the upper part of Figure 1.

The wireless channel is assumed multipath fading channel with N_p paths modeled by independent zero mean complex Gaussian processes $a_l(t) = \alpha_l(t) e^{j\phi_l(t)}$ $l = 0, ..., N_p - 1$ modeled using Jakes' model [8]. The fading rate of the channel is defined as the product $\omega_D T_s$, where ω_D is the maximum Doppler shift which depends on the system operating frequency and the velocity of the user. Neglecting the effect of the long term shadowing and path loss, the received signal at the input of the receiver has the following form:

$$r(t) = \sum_{l=0}^{N_p-1} a_l(t) s(t - \tau_l) + n(t) \tag{2}$$

where $\tau_l, l = 0, ..., N_p - 1$ are the path delays and $n(t)$ is zero mean complex AWGN with power spectral density N_0.

At the receiver of user i the signal is first sampled at the chip interval and the samples are fed into a RAKE receiver with L_p fingers. In each finger, the samples are multiplied by the complex conjugate of the corresponding channel coefficients (MRC combining), and then the real and imaginary parts are correlated with appropriately delayed versions of PN sequences $p_I(n)$ and $p_Q(n)$ respectively. The output of each finger is formed by adding the outputs of the I and Q correlators, and the output of the RAKE receiver by adding the outputs of all fingers. The symbol observations $y_i(k), k = 0, ..., N_f - 1$ at the input of the decoder after performing orthogonal despreading with the user's Walsh sequence have the following form:

$$\begin{aligned} y_i(k) = & \Big(\sum_{l=0}^{L_p-1} \alpha_l^2(k)\Big) \sqrt{2E_c}\, d_i(k) + 1/N_s \sum_{l=0}^{L_p-1} \alpha_l(k) \\ & \Big(\sum_{n=0}^{N_s} (p_I(kN_s + n - \tau_l) w_i(n) \eta_I(n) + \\ & p_Q(kN_s + n - \tau_l) w_i(n) \eta_Q(n))\Big) + ISI \end{aligned} \tag{3}$$

where $\eta_I(n), \eta_Q(n)$ are the real and imaginary parts of the AWGN noise samples (zero mean Gaussian with variance $N_0/2$), and ISI is the residual intersymbol interference caused by imperfect PN sequence autocorellation properties. In an ideal system the noise terms contributed by each finger in (3) would be independent and there would be no ISI term. Note that in (3), as well as in the rest of this paper, we assume that the channel coefficients are constant during one symbol duration T_s, although they may change over successive symbol intervals. The block diagram of the receiver is depicted in the lower part of Figure 1.

B. Concatenated Encoding and Iterative Decoding

In the forward link of the IS-95 standard [7] the channel encoder is convolutional of rate R=1/2 for rate I option with 9.6 kbps (or punctured to R=3/4 for rate II option with 14.4 kbps) and constraint length L=9 (memory $\nu = 8$, generators $f_0(D) = (753)_{oct}, f_1(D) = (561)_{oct}$), and the channel decoder uses the MLSE Viterbi algorithm. We replace the convolutional code by three different concatenated schemes employing interleavers: a parallel Turbo code as depicted in Figure 2(a), a serial Turbo code as shown in Figure 2(b), and a serially concatenated

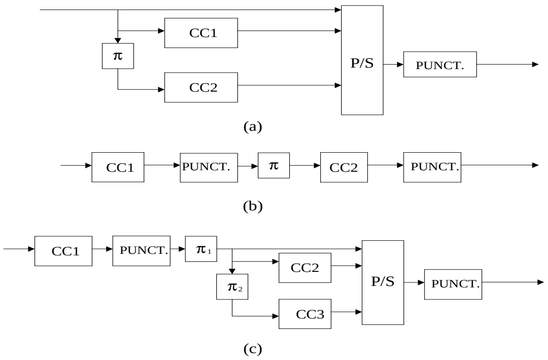

Figure 2: Concatenated encoding schemes.

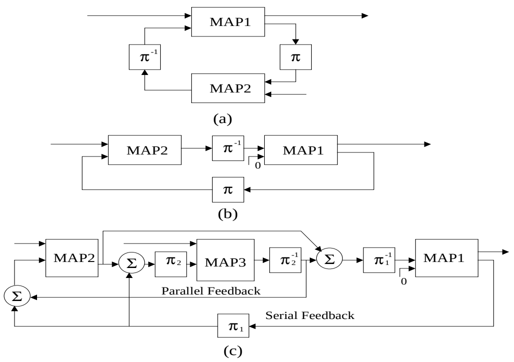

Figure 3: Iterative decoding schemes.

scheme with a convolutional outer code and a parallel Turbo inner code as shown in Figure 2(c). In Figure 2 we denote with CCi the component Convolutional Codes, and with π_i the random uniform interleavers. In the parallel Turbo case, both encoders are recursive systematic designed to optimize performance [9]. In the serial and the hybrid Turbo case, the outer encoder is non-recursive, non-systematic, maximum free distance code in order to maximize the interleaving gain [5], while the inner codes are recursive systematic as in the parallel scheme.

The main component of all the iterative decoders is the Maximum A-Posteriori Probability (MAP) module, which is based on the Log-MAP algorithm [10], a variation of the original BCJR algorithm [11]. Each MAP module accepts observations of the coded data and a-priori information about the uncoded data as inputs and produces Log-Likelihood Ratios (LLR's) and "extrinsic" information about the coded or uncoded data as outputs. The term extrinsic information denotes the incremental LLR information contributed by the MAP decoder.

The block diagram of the decoders we consider is depicted in Figure 3. In the parallel Turbo scheme MAP1 receives observations of the CC1 encoded bits from the channel and a-priori information from MAP2. It produces LLR's for the uncoded symbols which are used to estimate the data sequence, and extrinsic information which is passed as a-priori information to MAP2. MAP2 receives observations of the CC2 encoded symbols and a-priori information from MAP1 and produces extrinsic information which is passed as a-priori information to MAP1, and this completes the iteration.

In the serial Turbo scheme the inner MAP decoder (MAP2) receives observations of the CC2 encoded symbols and a-priori information from the outer decoder (MAP1). It produces extrinsic information for the CC2 input symbols which is passed as observation to MAP1. Note that when the inner code is systematic, the systematic channel observation is appropriately scaled and added as part of the extrinsic information passed as observation to the outer decoder. MAP1 receives no a-priori information. It produces LLR's which are used to estimate the data sequence, and extrinsic information which is passed as a-priori information to MAP2.

The hybrid scheme decoder is a combination of the parallel and serial Turbo decoders. Both inner decoders MAP2 and MAP3 receive a-priori information from the outer decoder MAP1 (serial feedback, outer iteration), in addition to exchanging information between them (parallel feedback, inner iteration). The outer decoder MAP1 has as observation the sum of the extrinsic information produced by the two inner decoders, and no a-priori information.

III. PERFORMANCE COMPARISONS

In this section we present simulation results on the performance of the different encoding schemes. In all cases we assume a channel with three independently fading paths with power ratios 2 and 4 dB and arrival delays $5T_c$ and $10T_c$ between the first and second, and first and third paths respectively. Furthermore, we assume that perfect channel estimates are available and that the RAKE receiver has three fingers that track continuously the three signal arrivals. The definition of average SNR per data binary symbol γ_b is derived from (3) by neglecting the correlation between the thermal noise contributions of the three RAKE fingers and the ISI term, although they are still present in the implemented system:

$$\gamma_b = \frac{\mathcal{E}(\sum_{l=0}^{N_p-1} \alpha_l^2) E_b}{N_0} \tag{4}$$

where $E_b = E_d/R$ is the energy per uncoded data symbol, and E_d is the energy per coded (transmitted) symbol, which is related to the energy per chip E_c according to $E_d = N_s E_c$. In the results that follow we normalize the channel coefficients such that $\mathcal{E}(\sum_{l=0}^{N_p-1} \alpha_l^2) = 1$.

We consider parallel Turbo codes (initial rate R=1/3, punctured appropriately) with recursive systematic component encoders with constraint lengths L=4 (memory

$\nu = 3$, [1, $(1+D+D^3)/(1+D^2+D^3)$] as proposed in the IS-95C physical layer draft) and L=5 (memory $\nu = 4$, [1, $(1+D^2+D^3+D^4)/(1+D+D^4)$] as proposed in [9]). In the case of serial Turbo codes the same recursive systematic codes are used as inner component codes, and R=1/2 convolutional codes with L=4 ($[1+D+D^3, 1+D+D^2+D^3]$) and L=5 ($[1+D^3+D^4, 1+D+D^2+D^4]$) as outer. They are punctured to R_{out}=3/4 and R_{out}=6/7, and R_{in}=2/3 and R_{in}=7/8 for overall R=1/2 and R=3/4 respectively. The hybrid Turbo scheme uses the same inner and outer component codes as the serial scheme. We choose to have 2 inner iterations occuring for every outer iteration. It is important to note that although in Figure 3 the hybrid Turbo decoder appears much more complex than the other two, using 5 outer iterations and 2 inner its computational complexity is only 25 % higher than a parallel or serial Turbo decoder using 10 iterations. In all cases we use the above number of iterations for iterative decoding.

The performance of the different schemes when the channel interleaver spans one frame (384 encoded symbols) is depicted in Figure 4 for coding rates R=3/4 (N=288) and R=1/2 (N=192). We notice that even for these small interleaving lengths the parallel Turbo scheme outperforms the convolutional (L=9) scheme by about 1 dB for a BER=10^{-3} and between 1 and 2 dB for a FER=10^{-2}. The serial Turbo has poorer BER and comparable FER performance with the parallel Turbo scheme, although its performance improves drastically for higher SNR's. This observation is in line with what was observed in AWGN channels [5], where it was shown that although in general the serial scheme is performing worse than the parallel at lower SNR's, it outperforms it in higher SNR's and doesn't display the error floor typical in parallel Turbo schemes. One last conclusion drawn from Figure 4 is that the parallel Turbo code with L=4 is only slightly worse than with L=5, making the use of an iterative MAP decoder with only 8 states a favorable alternative to using a 256 state Viterbi decoder.

The effect of the channel fading rate $\omega_D T_s$ on the performance of the convolutional and the parallel/serial Turbo schemes is depicted in Figure 5 for SNR=4 dB and code rate R=1/2. We notice that although, as expected for known channel, the performance of all schemes improves with higher fading rates, the improvement for both Turbo schemes is more dramatic, especially for high fading rates.

In Figure 6 we consider the case that a large block of data is encoded, interleaved, and then transmitted in multiple (M=5 frames/block) 20 *msec* wireless frames. For 20 *msec* frames this means that the decoding delay is 100 *msec*, which is acceptable for many applications. In Figure 6 we also plot the subFrame Error Rate for the convolutional code, assuming that each block is divided in M sub-frames, each with a separate CRC. Our experiments showed that the FER improvement in Turbo codes

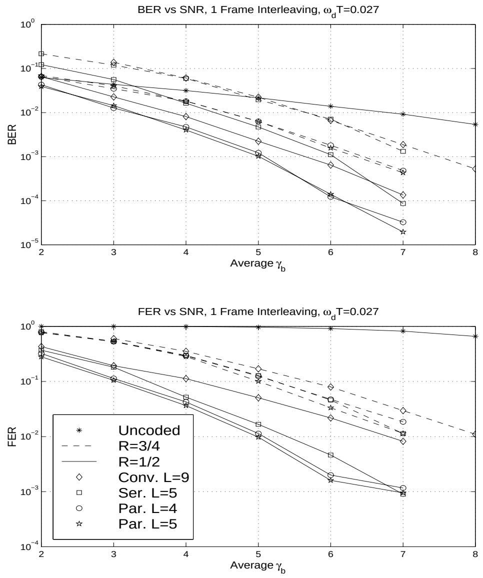

Figure 4: Performance comparisons for single (M=1) frame interleaving.

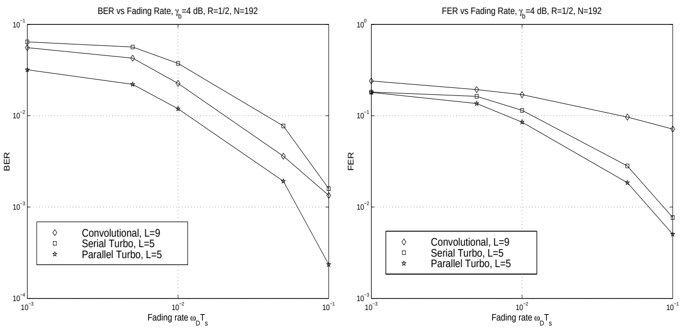

Figure 5: Effect of the fading rate $\omega_D T_s$.

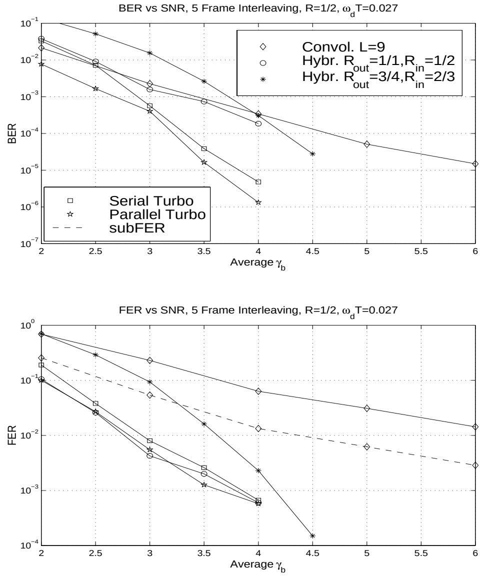

Figure 6: Performance comparisons for multiple (M=5) frame interleaving. L=4 for all Turbo schemes.

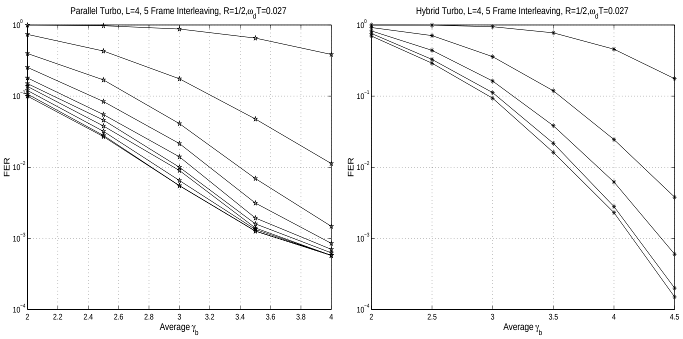

Figure 7: Effect of number of iterations on FER.

by adding CRC's is much smaller and does not justify the extra overhead. In Figure 6 we also present performance results of the hybrid scheme for two different configurations: in the first case, the punctured rate of the outer code is R_{out}=1/1 making the hybrid scheme perform more like a parallel Turbo, while in the second case the punctured rate of the outer code was chosen R_{out}=3/4. Bit errors in this system appeared very clustered giving it poorer BER performance when compared with the parallel and the serial Turbo schemes, although still better than the convolutional scheme. What is remarkable about the performance of the hybrid Turbo scheme, is that with R_{out}=3/4, the FER slope is much steeper than both of the other Turbo schemes, and especially the parallel Turbo scheme. This is also depicted in Figure 7 which shows the FER performance of the two systems for each iteration (outer loop iterations for the hybrid).

IV. CONCLUSIONS

In this paper we examined the application of three concatenated coding schemes to an IS-95-based CDMA system operating in a multipath Rayleigh fading channel. We investigated the effect of parameters such as code constraint length, interleaving length, and the channel fading rate. The results obtained show that even for relatively short interleaver lengths the concatenated schemes with L=4 outperform the convolutional code with L=9, particularly when FER is the figure of merit. Allowing interleaving over multiple frames or increasing the fading rate improves the performance of all schemes, and especially the Turbo ones.

Comparisons among the three Turbo schemes showed that the parallel scheme has the best performance for lower SNR's, but the slope of the performance curves for higher SNR's is steeper for the serial and hybrid schemes, which eventually outperform the parallel.

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